Well Behaved Search and the Robbins Problem

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Outline

- The Robbins Problem
- The Program EQP and Search Strategies
- The Robbins Solution
- Well Behaved Search

The Robbins Problem

- A well-known basis for Boolean algebra [Huntington, 1933]:

$$\begin{array}{ll} x+y=y+x & \text{(C) Commutativity} \\ (x+y)+z=x+(y+z) & \text{(A) Associativity} \\ \overline{(\overline{x}+y)}+\overline{(\overline{x}+\overline{y})}=x & \text{(H) Huntington} \end{array}$$

- The Robbins Problem:

$$\overline{(\overline{(x+\overline{y})}+\overline{(x+y)})}=x \quad \text{(R) Robbins}$$

  Is $\{C, A, R\}$ basis for Boolean algebra?

- A proof of the following would solve the problem.

  Robbins $\Longrightarrow_{\mathrm{AC}}$ Huntington.

Easy Lemmas About the Robbins Equation

A very easy one:

$$\left\{\begin{array}{l}\text{Robbins} \\ \forall x\ (\overline{\overline{x}}=x)\end{array}\right\} \Longrightarrow_{\mathrm{AC}} \text{Huntington.}$$

Another easy one (but not first-order):

$$\left\{\begin{array}{l}\text{Robbins} \\ \text{over a finite set}\end{array}\right\} \Longrightarrow_{\mathrm{AC}} \text{Huntington.}$$

Existence of a zero:

$$\left\{\begin{array}{l}\text{Robbins} \\ \exists x \forall y\ (x+y=y)\end{array}\right\} \Longrightarrow_{\mathrm{AC}} \text{Huntington.}$$

Idempotence:

$$\left\{\begin{array}{l}\text{Robbins} \\ \forall x\ (x+x=x)\end{array}\right\} \Longrightarrow_{\mathrm{AC}} \text{Huntington.}$$

Lemma 0. Existence of an idempotent element:

$$\left\{\begin{array}{l}\text{Robbins} \\ \exists x\ (x+x=x)\end{array}\right\} \Longrightarrow_{\mathrm{AC}} \text{Huntington.}$$

Hard Lemmas About the Robbins Equation

These were formulated and proved by Steve Winker c. 1980 [J. Algebra 1992].

Lemma 1 (An absorption):

$$\left\{ \begin{array}{l} \text{Robbins} \\ \exists x \exists y\ (x + y = x) \end{array} \right\} \Longrightarrow_{\text{AC}} \text{Huntington.}$$

Lemma 2 (An absorption within complement):

$$\left\{ \begin{array}{l} \text{Robbins} \\ \exists x \exists y\ (\overline{x + y} = \overline{x}) \end{array} \right\} \Longrightarrow_{\text{AC}} \text{Huntington.}$$

Winker's proofs are not entirely first-order (they use some induction).

EQP proved these lemmas automatically in 1992 and 1995.

EQP (EQuational Prover)

- Similar to unfailing KB completion.
- AC unification and matching.
- Term ordering: RPO (non-AC), ad hoc.
- Designed for fast rewriting:
  - some indexing for AC matching
  - fast AC matching
- Various search strategies:
  - unit of work – pair of equations or single equation;
  - basic paramodulation
  - AC unification restriction (super-0)
  - shortest-first vs. breadth-first
  - weight limits

EQP vs. Otter

Similarities

- performance (EQP is a bit faster)
- indexing and unification (excluding AC)
- I/O language and design philosophy (C)

Differences

- they share very little code
- Otter uses structure sharing of terms
- EQP has AC unification
- Otter accepts nonunit clauses

Term Ordering

A simple simplification ordering:

$$t_1 \succ t_2 \text{ if} \left\{ \begin{array}{c} \text{length}(t_1) > \text{length}(t_2) \\ \text{and} \\ \text{multiset-vars}(t_1) \supseteq \text{multiset-vars}(t_2) \end{array} \right.$$

- This is primitive, but all that was needed.
- Before the key proofs are found,
  - every derived equation is orientable,
  - almost always, t_2 is a AC-subterm of t_1.

Paramodulation with Extensions

Equation E: $a + b \quad = c$
Extension of E: $a + b + x = c + x$

- Extensions are sometimes necessary, and sometimes helpful.
- EQP always uses extensions right away.
- Others have found that it is better to delay use of extensions.
- But, extensions play a small role in the Robbins work.
  - Almost all equations have $complement$ at the root of the heavier side: $\overline{t_1} = t_2$.

Basic Paramodulation

- Do not paramodulate into terms that arise from instantiation.

$$\frac{f(a) = b, \quad j(f(x), x) = g(x)}{j(b, a) = g(a)}$$

- In the conclusion,
  - b is admissible for paramodulation (basic),
  - a is not admissible (nonbasic).
- This imposes an order on inferences and proofs.
- Basic inference is complete for several kinds of narrowing, superposition, and paramodulation.

Super-0 AC Unification

Mark Stickel's AC Unification Algorithm: t_1, t_2.

- set up a Diophantine equation representing identity of t_1 and t_2;
- find a basis of solutions;
- consider each subset of the basis:
  - if immediate constraints are satisfied we have a *potential unifier*;
  - if we have a potential unifier, attempt to unify any subterms.

The *super-0 restriction*: If a subset S gives a potential unifier, do not consider any supersets of S.

Super-0 AC Unification (cont'd)

- It is an incomplete heuristic.
  - In practice, it doesn't seem to block many proofs.
- It is quite severe:
  - The pair of terms

    $$x + x + x$$
    $$y + z + u + v$$

    has 1,044,569 AC unifiers.
  - With super-0, we get 139 AC unifiers.

Basic and Super-0 (Robbins Lemmas)

Lemma 0, Lemma 1, and Lemma 2 proofs.

	Lem 0	Lem 1	Lem 2
(neither)	4.5 sec	1.9 hr	7 days
Basic	3.9 sec	0.7 hr	>7 days
Super-0	2.3 sec	2.6 hr	6.5 days
Basic, Super-0	1.5 sec	3.9 hr	??

Basic and Super-0 (Robbins Theorem)

Robbins searches (without special hypotheses) after considering 5000 pairs of equations:

	Basic	Super-0	Super-0 Basic
Pairs of equations	< 5000*	5000	5000
Equations derived	??	66009	9497
Rewrite attempts	??	3,335,415	384303
Rewrites	??	81285	403
Forward subsumed	??	19764	200
Deleted by weight	??	37122	4506
Retained	??	9123	4791
Time	>4 days	24 hours	2 hours

* The search without Super-0 got stuck on a hard unification problem:

$$n(n(n(x+y)+n(x+y)+n(x)+y+y+y)+n(x+y))$$
$$n(n(n(n(n(x)+y)+x+y)+y+z)+n(n(n(x)+y)+z))$$

The Final Assault

- 5 weeks of real time, 5 weeks of CPU time.
- About 50 searches with various strategies, including 14 multiday searches.
  - Many small attempts to achieve well-behaved search
- Success: Robbins $\Longrightarrow_{\text{AC}} \exists x \exists y\ (\overline{x+y} = \overline{x})$
  - 8 day search, 30 megabytes of memory.
  - Basic paramod., super-0 AC restriction.
  - Small inference tasks.
  - Equal parts breadth-first, shortest-first search.
  - Discard equations with more than 70 symbols.
- Quicker proofs and shorter proofs were found later.

Solutions of the Robbins Problem

First proof (EQP, 8 days of search):

$$\text{Robbins} \Longrightarrow_{\text{AC}} \exists x \exists y\ (\overline{x+y} = \overline{x})$$

Another proof (EQP, a bit more search):

$$\text{Robbins} \Longrightarrow_{\text{AC}} \exists x \exists y\ (x+y = x)$$

Yet another proof (EQP, 10%–90% more search):

$$\text{Robbins} \Longrightarrow_{\text{AC}} \text{Huntington}$$

A Proof that Solves the Robbins Problem

Theorem A. *Robbins* $\Longrightarrow_{\mathrm{AC}} \exists c \exists d\ (c+d=c)$.

7	$\overline{\overline{\overline{x}+y}+\overline{x+y}} = y$	[Robbins Equation]
10	$\overline{\overline{\overline{x+y}+\overline{x}+y}+y} = \overline{x+y}$	[7 → 7]
11	$\overline{\overline{\overline{\overline{x}+y}+x+y}+y} = \overline{\overline{x}+y}$	[7 → 7]
29	$\overline{\overline{\overline{\overline{x}+y}+x+2y}+\overline{\overline{x}+y}} = y$	[11 → 7]
54	$\overline{\overline{\overline{\overline{\overline{x}+y}+x+2y}+\overline{\overline{x}+y}+z}+\overline{y+z}} = z$	[29 → 7]
217	$\overline{\overline{\overline{\overline{\overline{x}+y}+x+2y}+\overline{\overline{x}+y}+\overline{y+z}+z}+z} = \overline{y+z}$	[54 → 7]
674	$\overline{\overline{\overline{\overline{\overline{\overline{x}+y}+x+2y}+\overline{\overline{x}+y}+\overline{y+z}+z}+z+u}+\overline{\overline{y+z}+u}} = u$	[217 → 7]
6736	$\overline{\overline{\overline{\overline{3x}+x}+\overline{3x}}+\overline{\overline{\overline{3x}+x}+5x}} = \overline{\overline{3x}+x}$	[10 → 674]
8855	$\overline{\overline{\overline{3x}+x}+5x} = \overline{3x}$	[6736 → 7 :54]
8865	$\overline{\overline{\overline{\overline{3x}+x}+\overline{3x}+2x}+\overline{3x}} = \overline{\overline{3x}+x}+2x$	[8855 → 7]
8866	$\overline{\overline{\overline{3x}+x}+\overline{3x}} = x$	[8855 → 7: 11]
8870	$\overline{\overline{\overline{\overline{3x}+x}+\overline{3x}+y}+\overline{x+y}} = y$	[8866 → 7]
8871	$\overline{\overline{3x}+x}+2x = 2x$	[8865 :8870]

Equation 8871 asserts the existence of an object c, namely $2x$, and an object d, namely $\overline{\overline{3x}+x}$, such that $c+d=c$. Q.E.D. (Note that 8855 satisfies $\exists c \exists d(\overline{c+d}=\overline{c})$.)

Some Statistics on the Preceding Proof

(which is not the first proof)

- Length limit 50.
- 10 days (15MB RAM) on a SPARC-5 class computer.
- ~65,000 pairs of equations were considered.
  - $<1\%$ of total time, including AC unif.
  - ~125,000 equations (critical pairs) derived
- Rewriting was attempted on ~7,500,000 terms.
  - 95% of total time
  - ~18,000 terms rewritten
- 8871 equations were retained (all oriented).
  - 4% of time spent finding equations to back simplify
  - ~55 equations back simplified

Step 6736 in Detail

Premises:

$$\overline{\overline{\overline{x+y}+\overline{x}+y}+y} = \overline{x+y} \qquad (10)$$

$$\overline{\overline{\overline{\overline{\overline{\overline{x}+y}+x+2y}+\overline{\overline{x}+y}+\overline{y+z}+z}+z+u}+\overline{\overline{y+z}+u}} = u \qquad (674)$$

With (10), let x be $3v$, and y be $\overline{\overline{3v}+v}+2v$:

$$\underbrace{\overline{\overline{\overline{3v+\overline{\overline{3v}+v}+2v}+\overline{3v}+\overline{\overline{3v}+v}+2v}+\overline{\overline{3v}+v}+2v}} = \overline{3v+\overline{\overline{3v}+v}+2v} \qquad (10')$$

With (674), let x be $3v$, y be v, z be $2v$, and u be $\overline{\overline{3v}+v}$:

$$\overline{\underbrace{\overline{\overline{\overline{\overline{3v}+v}+5v}+\overline{\overline{3v}+v}+\overline{3v}+2v}+2v+\overline{\overline{3v}+v}}+\overline{\overline{3v}+\overline{\overline{3v}+v}}} = \overline{\overline{3v}+v} \qquad (674')$$

Replace the braced term of (674′), which is AC-identical to the left-hand side of (10′), with the right-hand side of (10′):

$$\overline{\overline{3v+\overline{\overline{3v}+v}+2v}+\overline{\overline{3v}+\overline{\overline{3v}+v}}} = \overline{\overline{3v}+v}.$$

Rename the variable and rearrange:

$$\overline{\overline{\overline{\overline{3x}+x}+\overline{3x}}+\overline{\overline{\overline{3x}+x}+5x}} = \overline{\overline{3x}+x}. \qquad [10 \to 674]\ (6736)$$

Lemma 0: $\left\{ \begin{array}{l} \text{Robbins} \\ \exists x\ (x+x=x) \end{array} \right\} \Longrightarrow_{\mathrm{AC}}$ Huntington.

1	$c+c=c$	[hypothesis]
2	$\overline{\overline{\overline{x}+y}+\overline{x+y}} = y$	[Robbins]
3	$\overline{B+\overline{A}}+\overline{\overline{B}+\overline{A}} \neq A$	[denial of Huntington]
4	$\overline{\overline{c}+\overline{c+\overline{c}}} = c$	[1 → 2]
5	$\overline{\overline{c+\overline{c}+x}+\overline{c+x}} = c+x$	[1’ → 2]
8	$\overline{\overline{c+x}+\overline{\overline{c}+\overline{c+\overline{c}}+x}} = x$	[4 → 2]
9	$\overline{c+\overline{c+\overline{c}+\overline{c}}} = \overline{c}$	[4 → 2]
13	$\overline{\overline{c}+\overline{c+\overline{c}+\overline{c+\overline{c}}}} = c$	[1 → 8]
20	$\overline{\overline{c}+\overline{c+\overline{c}+\overline{c}}} = c$	[9 → 2:1]
22	$\overline{c+\overline{c}+\overline{c}} = \overline{c+\overline{c}}$	[9 → 2:20]
24	$\overline{c+\overline{c+\overline{c}}} = \overline{c}$	[9:22]
32	$c+\overline{c+\overline{c}} = c$	[24 → 5:13]
35	$\overline{c+\overline{c}}+x = x$	[32’ → 2:8]
42	$\overline{\overline{\overline{x}}+\overline{x}} = \overline{c+\overline{c}}$	[35 → 2:35]
50	$\overline{\overline{\overline{\overline{x}}+x}} = \overline{\overline{x}}$	[42 → 2:35]
52	$\overline{\overline{\overline{x}}} = \overline{x}$	[42 → 2:35,50]
58	$\overline{\overline{x}} = x$	[2 → 52:2]
87	$\overline{\overline{x}+y}+\overline{x+y} = \overline{y}$	[2 → 58]
88	$A \neq A$	[3:87,58]

Q.E.D. (in a few seconds)

Lemma 1: $\left\{\begin{array}{l}\text{Robbins}\\ \exists x\exists y\ (x+y=y)\end{array}\right\} \Longrightarrow_{\text{AC}} \exists x\ (x+x=x).$

1	$x+x\neq x$	[denial of conclusion]
2	$d+c=c$	[hypothesis]
3	$\overline{\overline{\overline{x}+y}+\overline{x+y}}=y$	[Robbins]
4	$\overline{\overline{c}+\overline{d+\overline{c}}}=d$	[2 → 3]
8	$\overline{\overline{d+\overline{c+x}+y}+\overline{c+x+y}}=d+y$	[2' → 3]
20	$\overline{d+\overline{c+\overline{d+\overline{c}}}}=\overline{d+\overline{c}}$	[4 → 3]
34	$\overline{\overline{\overline{\overline{x}+y}+\overline{x+y}+z}+\overline{y+z}}=z$	[3 → 3]
35	$\overline{\overline{\overline{\overline{x}+y}+x+y}+y}=\overline{\overline{x}+y}$	[3 → 3]
56	$\overline{\overline{c}+\overline{d+\overline{c+\overline{x}}+\overline{c+x}}}=d$	[2 → 34]
151	$\overline{\overline{d+\overline{c+\overline{d+\overline{c}}}+x}+\overline{\overline{d+\overline{c}}+x}}=x$	[20 → 3]
152	$\overline{\overline{d+\overline{c}}+\overline{c+\overline{d+\overline{c}}}}=d$	[20 → 3:2]
173	$\overline{d+\overline{d+\overline{c}+\overline{c+\overline{d+\overline{c}}}}}=\overline{c+\overline{d+\overline{c}}}$	[152 → 3]
197	$\overline{\overline{c+\overline{d+\overline{c}}}+\overline{c+\overline{c+\overline{d+\overline{c}}}}}=c$	[2' → 151]
280	$\overline{\overline{\overline{\overline{\overline{x}+y}+x+y}+y+z}+\overline{\overline{\overline{x}+y}+z}}=z$	[35 → 3]
837	$\overline{c+\overline{d+\overline{c}}}=\overline{c}$	[4 → 151:173]
839	$\overline{\overline{c}+\overline{c+\overline{c}}}=c$	[197:837,837]
842	$\overline{\overline{c+x}+\overline{\overline{c}+\overline{c+\overline{c}}+x}}=x$	[839 → 3]
844	$\overline{c+\overline{c+\overline{c+\overline{c}}}}=\overline{c+\overline{c}}$	[839 → 3]
883	$\overline{\overline{c+\overline{c}}+\overline{c+c+\overline{c+\overline{c}}}}=c$	[844 → 3]
946	$\overline{c+\overline{c+\overline{c}+\overline{c+c+\overline{c+\overline{c}}}}}=\overline{c+c+\overline{c+\overline{c}}}$	[883 → 3]
1706	$\overline{c+c+\overline{c+\overline{c}}}=\overline{c}$	[839 → 280:946]
1734	$d+\overline{c+\overline{c}}=d$	[1706 → 8:56]
1745	$c+\overline{c+\overline{c}}=c$	[2' → 1734':2]
1802	$\overline{c+\overline{c}}+x=x$	[1745' → 3:842]

Q.E.D. (in a few hours)

Lemma 2: $\left\{\begin{array}{l}\text{Robbins}\\ \exists x\exists y\ (\overline{x+y}=\overline{y})\end{array}\right\} \Longrightarrow_{\text{AC}} \exists x\exists y\ (x+y=x).$

1	$x+y\neq x$	[denial of conclusion]
2	$\overline{d+c}=\overline{c}$	[hypothesis]
3	$\overline{\overline{\overline{x}+y}+\overline{x+y}}=y$	[Robbins]
4	$\overline{\overline{d+c+x}+\overline{\overline{c}+x}}=x$	[2 → 3]
5	$\overline{\overline{c}+\overline{d+\overline{c}}}=d$	[2 → 3]
7	$\overline{\overline{d+x}+\overline{\overline{c}+\overline{d+\overline{c}}+x}}=x$	[5 → 3]
9	$\overline{d+\overline{c+\overline{d+\overline{c}}}}=\overline{d+\overline{c}}$	[5 → 3]
18	$\overline{\overline{\overline{\overline{x}+y}+x+y}+y}=\overline{\overline{x}+y}$	[3 → 3]
25	$\overline{\overline{d+c+\overline{\overline{c}+x}+x}+x}=\overline{\overline{c}+x}$	[2 → 18:2]
35	$\overline{\overline{\overline{d+c+x}+\overline{\overline{c}+x}+y}+\overline{x+y}}=y$	[4 → 3]
64	$\overline{\overline{d+\overline{c}}+\overline{d+c+\overline{d+\overline{c}}}}=d$	[9 → 3]
141	$\overline{\overline{\overline{\overline{x}+y}+x+y+y}+\overline{\overline{x}+y}}=y$	[18 → 3]
145	$\overline{\overline{\overline{\overline{x}+y}+\overline{x+y}+\overline{y+z}+z}+z}=\overline{y+z}$	[3 → 18:3]
170	$\overline{d+\overline{d+\overline{c}+\overline{d+c+\overline{d+\overline{c}}}}}=\overline{d+c+\overline{d+\overline{c}}}$	[64 → 3]
287	$\overline{\overline{\overline{\overline{\overline{x}+y}+x+y+y}+\overline{\overline{x}+y}+z}+\overline{y+z}}=z$	[141 → 3]
298	$\overline{d+c+\overline{d+\overline{c}}}=\overline{c}$	[5 → 35:5,170]
826	$\overline{\overline{d+\overline{c}}+\overline{d+\overline{d+\overline{c}}+\overline{\overline{c}+\overline{x}}+\overline{\overline{c}+x}}}=d$	[5 → 145:5]
1234	$\overline{\overline{\overline{\overline{\overline{x}+y}+\overline{x+y}+\overline{y+z}+z}+z+u}+\overline{\overline{y+z}+u}}=u$	[145 → 3]
1475	$\overline{d+d+c+\overline{d+\overline{c}}}=\overline{c}$	[25 → 7:287]
1498	$\overline{\overline{d+d+c+\overline{d+\overline{c}}+\overline{\overline{c}+x}+x}+x}=\overline{\overline{c}+x}$	[1475 → 18:1475]
3318	$\overline{d+\overline{d+\overline{c}+\overline{d+\overline{d+\overline{c}}+\overline{\overline{c}+\overline{x}}+\overline{\overline{c}+x}}}}=\overline{c}$	[5 → 1234]
3707	$\overline{d+\overline{d+\overline{c}}+\overline{\overline{c}+\overline{x}}+\overline{\overline{c}+x}}=\overline{c}$	[826 → 35:298,5,3318]
3708	$\overline{d+\overline{d+\overline{c}}+\overline{d+c+\overline{c}}+\overline{\overline{c}+\overline{c}}}=\overline{c}$	[2 → 3707]
3733	$d+\overline{d+\overline{c}}+\overline{\overline{c}+\overline{c}}=\overline{\overline{c}+\overline{c}}$	[3708 → 3:1498]

Q.E.D. (in a few days) This lemma is not needed for the overall proof.

All in One Proof

Theorem B: Robbins $\Longrightarrow_{\text{AC}}$ Huntington.

7	$\overline{\overline{\overline{x}+y}+\overline{x+y}}=y$	[Robbins]
8	$\overline{\overline{B+\overline{A}}+\overline{\overline{B}+\overline{A}}}\neq A$	[denial of Huntington]
10	$\overline{\overline{\overline{x+y}+\overline{x}+y}+y}=\overline{x+y}$	[7 → 7]
	(85 lines omitted)	
13555	$A\neq A$	[8:13546,13516]

Q.E.D. (in a few days) The complete proof is on the Web.

- This is from a continuation of the search that proves Theorem A:

$$\text{Robbins} \Longrightarrow_{\text{AC}} \exists c\exists d\ (c+d=c).$$

- But, using Lemma 0, Lemma 1, and Theorem A gives a simpler overall proof.

Well Behaved Search

A Simple but Practical Example

- Without a size limit on equations,
  - too many are retained,
  - wasting time and memory.
- With a size limit,
  - incompleteness,
  - the program can exhaust the search space,
  - not enough variety in the search.
- It can be difficult to find a good limit.
  - Minimum might not be best.
  - Restart search with a new limit?
  - Change limit during the search?

Bad Search/Good Search

Bad Search

- Wasting resources:
  - too much time rewriting or subsuming
- Uniform search:
  - all retained clauses look similar
- Getting stuck on a prolific equation.

Good Search

- Syntactic variation of derived equations
- Balance various properties

Some Properties of Search

- forward/backward
- positive/negative
- variable/ground
- unit/nonunit
- shortest-first/breadth-first
- complete/size limits
- uniform/varied
- term ordering and definitions: fold/unfold
- focus: small/large
- time spent in various operations

Achieving Well Behaved Search
Some Examples

- Observe derived equations, try to balance search properties, iterate.
- Compare results of different searches.
  - If a search is more economical and it gives a similar set of equations, use it.
- Restart the search with new lemmas.
  - Clean out the deadwood and focus on the lemmas.
- The Future
  - Self analytical theorem provers (Larry Wos).
  - A formal language for expressing notions of well behaved search (Lincoln Wallen).
  - Metareasoning.

Concluding Questions

- Is the proof correct?
- Why did EQP succeed where humans had failed?
- Are there many other problems in this class?
- How much further can we go with automatic theorem proving?
- How far can we go with interactive/automatic systems?

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