Formation of Stripe Domains in Cobalt Bars via a Magnetic Soft Mode Instability

M. Yan, G. Leaf, H. Kaper, V. Novosad, P. Vavassori, R. E. Camley, and M. Grimsditch

***Abstract*— We theoretically study the relation between stripe domains at remanence and magnetic normal modes in a single crystal Co bar. We find different stripe patterns depending on field history and in each case the domain structure can be related to a soft mode that triggers a phase transition. The stripe domain structure when the external field is along the long axis of the bar is shown to be generated by a standing wave mode, which has the same spatial structure as the stripes. At all fields this mode has the lowest frequency of all the standing wave modes. This mode goes soft at a second order phase transition where the stripe domains emerge. For other directions of the field, the symmetry of soft modes is found to be consistent with the change in symmetry of the ground state and that the phase transition can be first order. An analytical model relating phase transitions and soft mode behavior is also briefly discussed.**

Index Terms—**micromagnetics, phase transitions, soft modes, stripe domains**

I. Introduction

Stripe domains are common phenomena observed in many different physical systems [1]-[4]. Landau and Lifshitz first predicted the existence of stripe domains in ferromagnetic samples based on energy minimization consideration, since the formation of stripe domains can reduce the surface charge [5]. It remains unclear, however, how a system evolves into a particular stripe domain structure.

Here we investigate this issue by studying the magnetic normal modes of a thin-film cobalt bar with uniaxial anisotropy in the width direction of the bar, which is known to develop stripe domains at remanence [6]. In this paper, we first show that the stripe domain structure at remanence in such a system is not unique and depends on the field history. Hence energy minimization alone is not adequate to predict the exact stripe domains at remanence. To identify the dynamic origin of the stripe domains, we calculate the magnetic normal modes using micromagnetic-based technique [7], [8]. We focus on the simplest case in this paper when the external field is applied along the length of the bar. In this case, we find that one particular standing wave mode, that has the same spatial pattern as the stripe domains, always has the lowest frequency of all the standing wave modes, independently of the external field. This mode is found to go soft, *i.e*., approach zero frequency, at a continuous phase transition and nucleates the form of the stripe domains. A brief description will be given for another two cases with the external field applied along the width direction of the bar. In addition, an analytical model will be briefly discussed about how the soft mode profile determines the change in magnetization at the onset of a phase transition. The critical exponent of the soft mode frequency is also derived.

II. Results and discussion

In our numerical studies, we apply two methods to calculate the magnetic normal modes. One method is based on micromagnetic simulations and Fourier transform [7]. The other method involves a dynamical matrix approach [8]. Both methods enable us to obtain the frequency and profile of the normal modes.

We study a single crystal *hcp* Co bar, 40 nm thick and 120 nm wide, with a uniaxial anisotropy along the width direction, and a length of either 792 nm or 936 nm. The material parameters used in calculations are the saturation magnetization $Ms=1.4 \times 10^5$ A/m, the exchange constant $A = 3 \times 10^{-11}$ J/m, and the uniaxial anisotropy constant $K_u = 5.2 \times 10^5$ J/m^3; all are typical values for epitaxial cobalt films. For such a system, we obtain different stripe domains for different field histories. Below we describe three processes and the key outcomes.

1) A large field is applied along the long axis of the bar. As the field is reduced, the 792 nm bar breaks into 11 stripe domains. Similarly, the 936 nm bar breaks into 13 domains.
2) A large field is applied along the short axis of the bar, along the uniaxial direction. As the field is reduced and then made negative, the 792 nm bar breaks into 5 domains. If the magnetic field is

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then reduced to zero, the 5-domain state remains stable.

3) A large field is applied at an angle to the short axis of the bar. As the field is reduced and then made negative, the 792 nm bar breaks into 3 domains. Again, this state is stable at zero field.

The three stripe domains at remanence are shown in Fig. 1. The fact that the stripe domain structure at remanence is not unique implies the failure of energy minimization to predict the exact domain structure at remanence. Using simulations to find the ground state energy of a bar with an arbitrary number of domains, we find that none of the three remanent stripe domains listed above is at the lowest energy state at zero field.

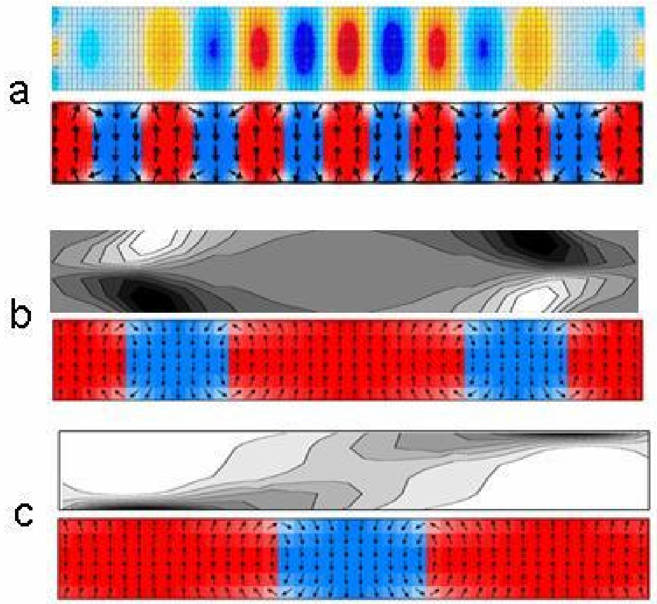

Figure 1 (color online) Comparison between the profiles of modes that go soft at the phase transition points (up) and stripe domains at remanence (below) in three different cases: (a) external field applied along the long axis for the 936 nm bar; (b) and (c) external field applied along the short axis and at an angle to the short axis respectively for the 792 nm bar.

We now focus on the formation of stripe domains for the case that the external field is applied along the long axis of the bar. More detailed results are published in Ref. [9]. As the field is reduced, magnetization reversal is a complex process initiated at the particle ends. However, simulations on bars that differ only in length show that the domain size at remanence is independent of the bar length, indicating that the final state is a bulk-like phenomenon rather than an "end effect."

The frequencies of the normal modes give us additional insight into the nature of the phase transition. In this case, the bulk modes resemble standing wave like solutions. In Fig. 2, we plot the frequency of the standing wave modes as a function of the wavevector, *q*, for three cases: the long bar at $H = 7kOe$, the long bar at $H = 6kOe$, and the short bar at $H = 6kOe$. The important point is that the lowest frequency mode is always the one with $q = 0.043nm^{-1}$ independent of length of the bar or the value of the field. This wavevector corresponds to a wavelength of about 146 nm. In both the long and short bar, we find that the average domain size is 72 nm. Since one wavelength should correspond to two domains, one wavelength would be 144 nm, in excellent agreement with the wavelength of the lowest-frequency standing wave mode. A "snapshot" of the out of plane component of the precessing magnetization for the lowest-frequency mode is shown in Fig. 1a at $H = 12kOe$ to compare with the domain pattern at remanence where $H = 0$. Clearly, the wavelength of this mode closely matches the typical domain size observed in the 13-domain state found in the micromagnetic simulations.

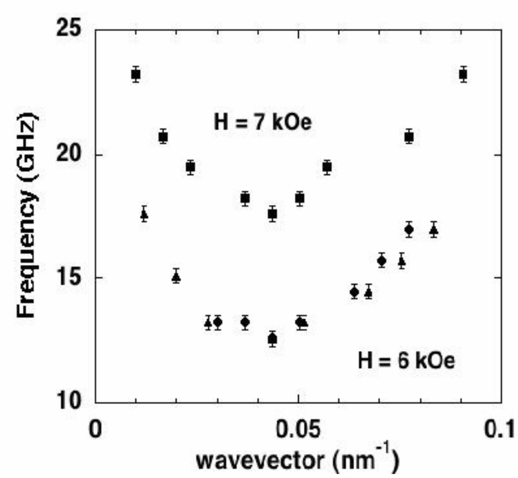

Figure 2 Frequency as a function of wavevector for standing-wave modes in both the long (936 nm, squares and circles) and short (792 nm, triangles) bars and for two different magnetic fields.

In Fig. 3 we plot the frequency of the lowest-frequency bulk mode (the one shown in Fig. 1a) as a function of field. For comparison, the frequency of the uniform mode is also shown. Although there are many other modes not shown in this figure, there is only one bulk mode that reaches zero frequency near $H = 5kOe$. Thus, in this geometry, the final state configuration is directly connected with a particular bulk mode that goes soft. Figure 3 also shows the evolution of an order parameter as a function of applied field. The squares in Fig. 3 are the maximum value of $|M_z|$ in the cells in the central portion of the bar where *z* is the short axis; as such, it provides a convenient order parameter for the phase transition. Noting that the order parameter and the frequency of the lowest bulk mode extrapolate to zero at the same field and the identical symmetry between the mode profile and stripe domains, we claim that the origin of the stripe domains in this geometry is a soft magnon mode, and not the minimization of energy.

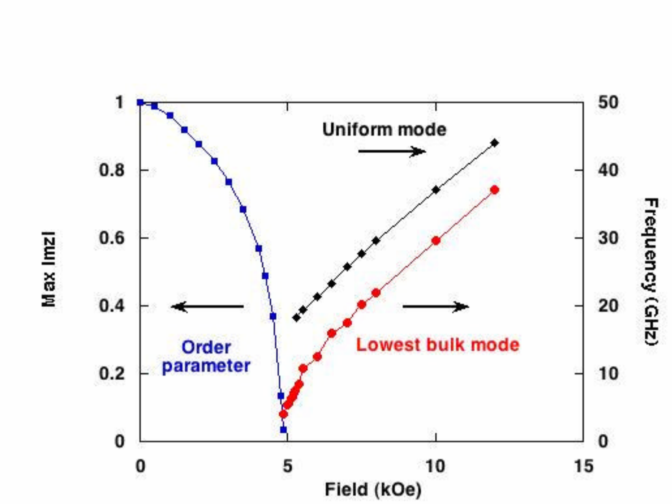

Figure 3 (color online) Frequency as a function of applied magnetic field for the lowest frequency bulk mode and the uniform resonance mode (right axis). The lefthand side shows the evolution of the "order parameter" $|M_z|$ in the central region of the bar.

In two other cases with the field applied along the short axis, the bar breaks into either 3 or 5 domains. The difference here is that the magnetization goes through a discontinuous change at a critical field and thus a first order phase transition occurs. Soft modes at the critical fields are observed as well. More interesting, the symmetry of the soft modes is consistent with that of the change of ground states. The spatial profiles of the soft modes and the corresponding final ground states are shown in Fig. 1b and c. The detailed results will be published elsewhere [10]. In all three cases, soft modes are responsible for the phase transitions observed in this system. Even more general is the fact that the profile of a soft mode determines the type of nucleation at that critical field. This applies equally to the formation of localized end domains or to the nucleation of the state that leads the system to develop stripe domains at remanence. In all cases this nucleation path need not be the lowest energy state at zero field.

An analytical model based on the dynamical matrix approach provides a physical picture how the soft mode profile determines the magnetization profile after the symmetry break in a phase transition. In the dynamical matrix approach, for a given equilibrium magnetization, the normal modes are obtained by solving an eigen equation, with the eigenvalue proportional to the frequency and the eigenvector yielding the mode profile. The matrix elements of the eigen equation can be shown to be proportional to the Hessian matrix. Since the Hessian matrix also provides a measure of the change in energy around the equilibrium configuration, it is easy to see that if a soft mode exists, the energy of the system is invariant for a configurational change proportional to the eigen vector of the soft mode. Conversely, since these configurational changes produce no change in energy, they are directly related to an instability. In the case of thin-film samples, the eigen function can be simplified under the assumption that all ground states of the system lies in the film plane. At any phase transition, it can be shown that the change in magnetization just after the symmetry break must be proportional to the soft mode eigenvector. When the transitions are second order, the critical exponent of the soft mode frequency can be shown to be equal to ½. This conclusion thus applies for the formation of stripe domains in the case of external field along the long axis of the bar and other phase transitions reported in Ref. [11]. In first order phase transitions, the relationship of symmetry between the soft mode and change of ground states can be qualitatively understood by considering the energy behavior of the system near the critical field. At the critical field, the total energy of the system becomes a saddle point in the configuration space. So that again, at the onset of the transition, the change of magnetization is proportional to the soft mode eigenvector. The details of this model will be published elsewhere [12].

In summary, we have studied the dynamic origins of stripe domains in a Co bar. We find that soft modes are responsible for all the phase transitions observed in the system, whether first order or second order. We also have developed an analytical model to describe the relation between soft modes and phase transitions.

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