Basic Research Problems:

The Problem of Automated Theorem Finding*

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Abstract. This article is the twenty-fifth of a series of articles discussing various open research problems in automated reasoning. The problem proposed for research asks one to identify appropriate properties to permit an automated reasoning program to *find* new and interesting theorems, in contrast to *proving* conjectured theorems. Such programs are now functioning in many domains as valuable reasoning assistants. A sharp increase in their value would occur if they could also be used as colleagues to (so to speak) produce research on their own.

Key words. Automated reasoning, theorem finding, unsolved research problem.

Question: *What properties can be identified to permit an automated reasoning program to find new and interesting theorems, as opposed to proving conjectured theorems?*

(This question is the thirty-first of 33 problems proposed for research in [3] and will be referred to as Research Problem 31 throughout this article. All references to sections, chapters, test problems, and such also refer to [3].)

The field of automated reasoning is an outgrowth of the field of automated theorem proving. In fact, the dominant activity in automated reasoning is still that of proving some (conjectured) theorem. Research Problem 31 asks for criteria that an automated reasoning program can use to find interesting theorems, in contrast to proving conjectured theorems supplied by the user. In particular, since a reasoning program can be instructed to draw some (possibly large) set of conclusions, Research Problem 31 asks for criteria that permit the program to select from those conclusions the ones (if any) that correspond to interesting results.

The test for a solution to Research Problem 31 rests with the expert in the field under investigation by the reasoning program. The proposed solution must enable a reasoning program to select results that, for example, the group theorist evaluates as interesting theorems. Obviously, solving this problem would be significant because of the assistance that the corresponding reasoning program could provide for researchers in various fields. To experiment with notions for solving this problem, one might find most useful W. McCune's automated reasoning program OTTER; a copy on diskette is offered in [4]. Although clearly not in the precise spirit of theorem finding, McCune used UNIX to generate candidate formulas, each of which was then submitted to OTTER with the goal of finding single axioms for group theory [1, 2]. The reason his excellent research is not precisely theorem finding is that our intention is to have the program select, from among its myriad of deduced conclusions, those that correspond to interesting results. In other words, the burden for identifying (deduced) results as worthy of the term theorem rests with the automated reasoning program, and not with the researcher possibly aided by UNIX or the like.

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*This work was supported by the Applied Mathematical Sciences subprogram of the Office of Energy Research, U.S. Department of Energy, under Contract W-31-109-Eng-38.

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