Basic Research Problems:

The Problem of Reasoning by Analogy*

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Abstract. This article is the twenty-seventh of a series of articles discussing various open research problems in automated reasoning. The problem proposed for research asks one to find criteria that an automated reasoning program can apply to determine whether to attack a given question with reasoning by analogy. The imprecise term 'reasoning by analogy' refers to a type of reasoning in which the type of proof being sought is sharply influenced by the style of proof that was successfully used to prove related theorems.

Key words. Automated reasoning, reasoning by analogy, representation, unsolved research problem.

Question: *Bearing in mind that reasoning by analogy is only one of many types of reasoning that people use to answer questions, what criteria can be applied to a question to cause an automated reasoning program to choose wisely to employ this type of reasoning?*

(This question is the thirty-third of 33 problems proposed for research in [4] and will be referred to as Research Problem 33 throughout this article. All references to sections, chapters, test problems, and such also refer to [4].)

One form of argument commonly used by people is that of argument by analogy. The use of this form of argument, although it is accepted as lacking rigor, often leads to uncovering important information. In particular, if the argument leads to interesting conclusions, one can then use that argument to guide the search for a corresponding rigorous argument. Research Problem 33 asks for criteria that a reasoning program can employ to decide to 'reason by analogy'. If such reasoning leads to a potentially valuable conclusion, the reasoning program could then attempt to find a rigorous argument proving the potentially valuable conclusion valid—a rigorous argument suggested by the analogy. (For an excellent review of analogy in automated theorem proving, see [3].)

In private conversation with Woody Bledsoe, we have learned that his research in proof by analogy is closely related to the discussion of the preceding paragraph (for a discussion of the use of 'scripts' to further the research on proof by analogy, see [1]; for a description of the development of an analogy-based prover, see [2]). In Chapter 10 of [5], we used the phrase 'proof by analogy' in a more restricted fashion, focusing on using the steps of the proof of one theorem to guide the search for a proof of another, related theorem. In that chapter and in Chapter 9, we show how, if the general mechanism of weighting [5] is used to implement as a strategy our restricted use of proof by analogy, satisfying results are obtained, including those concerned with finding shorter proofs. Because, for us, proof by analogy is of greatest interest when studied in the cited, narrower context, and because in that context we focus on its *strategic* aspects, we now use the term 'resonance strategy' [6].

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Since we have included Research Problem 33 as suggested research, it seems natural to suggest a somewhat related problem. Where the first focuses on reasoning, the second problem focuses on representation. Specifically, what criteria can a reasoning program employ to classify some given set of clauses as awkward or minimally acceptable and, having made such a classification, then replace the given set with a logically equivalent set possessing satisfactory properties? For example, a study of set theory might begin with axioms characterizing naive set theory. This beginning might be minimally acceptable because of its intuitive appeal. However, when the automated reasoning program is given the assignment of proving deep mathematical theorems, the naive set theory would then be replaced with Gödel's finite axiomatization (see Section 6.6).

A solution to the primary problem of reasoning by analogy would enhance the reasoning power of future programs by providing yet one additional means of attacking assignments. A solution to the related problem of representation could enable a reasoning program to replace undesirable sets of input clauses with desirable sets, resulting in an increase in effectiveness.

References

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