Glassy Motion of an Elastic String

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Abstract

Numerical simulations of a driven elastic string in a quenched random potential at finite temperatures support the existence of glassy motion for sufficiently weak driving forces. Avalanche-like string motion is observed in the transition region.

The dynamics of an elastic string in a random medium provides an excellent model for the description of a variety of interesting phenomena—including fluctuations of domain walls in random magnets [1], surface growth in a random environment [2], and directed polymer growth [3]. It is also extremely useful for the description of vortex motion in high-T_c superconductors. The elastic-string model is exact for vortex dynamics at moderate temperatures and magnetic fields, when the characteristic pinning barriers for isolated vortex motion exceed the energy associated with vortex-vortex interactions [4]. Moreover, when vortex-vortex interactions are significant, the ideas of elastic-string dynamics [5] can be generalized to describe the collective motion of vortex bundles [6].

In this communication we present some results of numerical simulations of the motion of an elastic string in random quenched disorder. The string is driven by a constant force and subject to thermal noise. The inclusion of both quenched random disorder and thermal noise distinguishes our model from those considered in interface dynamics, where the disorder is modeled with either thermal-like noise [2, 3, 7] or quenched random disorder [8]. Another distinction is that we consider motion under the influence of a *random potential*, rather than a *random force*. In a random potential, the fluctuations of the string energy are determined by the instantaneous position of the string; in the presence of a random force, the fluctuations are determined by the entire area swept by the string during its motion. Because the energy of a material defect is the same before and after interaction with a vortex, a description in terms of a random potential is appropriate for vortex dynamics in high-T_c superconductors. The dynamics of an elastic string subject to quenched disorder at finite temperature has recently been used for a numerical study of the behavior of the critical current [9].

We consider the motion of an overdamped elastic string of length L ($L \gg 1$), which is constrained to lie in a plane in physical space ($1+1$-dimensional case). The position of a point on the string is given by the pair of coordinates (x, u), where x is the coordinate along the string at rest ($0 < x < L$) and u the transverse displacement; u is a single-valued function of position x and time t. We assume that the string is extended periodically to the entire interval $-\infty < x < \infty$ at all times. The motion of the string is governed by a Langevin equation, $\Gamma \partial_t u = -\delta H / \delta u + f$, where H is the Hamiltonian and f the thermal force per unit length acting on the string; Γ is the damping coefficient. We render the problem dimensionless by taking the coherence length ξ of a pinning center as the unit of length and a reference energy E_0 as the unit of energy, measuring time in units of $\Gamma\xi^2/E_0$, temperatures in units of $\xi E_0/k_B$, and forces in units of E_0/ξ. Thus, the equation of motion of the string is

$$\partial_t u = C\partial_x^2 u - \partial_u V + F + f, \qquad 0 < x < L,\ t > 0. \tag{1}$$

Here, C is the coefficient of linear tension, F the (constant) driving force per unit length, and V the potential due to quenched disorder. We assume that the pinning centers that contribute to the potential V are identical and distributed randomly throughout the plane with a specified density ρ. Each pinning center is characterized by a radially symmetric potential with maximal strength U_p at the center. We take f to be Gaussian with zero average and temperature T,

$$< f(x,t) > = 0, \quad < f(x,t), f(x',t') > = 2T\delta(x-x')\delta(t-t'). \tag{2}$$

The quantity of interest is the *average string velocity* (i.e., the ensemble average of the local normal velocity averaged over the length of the string) at large times. If we replace the ensemble average by a time average, this quantity is given by

$$v = \lim_{t\to\infty} \frac{1}{t}\int_0^t \frac{1}{L}\int_0^L \partial_s u(x,s)\ dx\ ds. \tag{3}$$

In particular, we are interested in the functional dependence of v on the driving force F. If the elastic string models vortices in a high-T_c superconductor, then the computation of the graph of v vs. F corresponds to an experimental measurement of the I–V curve of a superconducting sample; F is proportional to the applied current I, v to the resulting voltage V.

The statistical mechanics of a $1+1$-dimensional elastic manifold in a random medium is now well understood. Rigorous results have been obtained for disorder-induced roughening [1, 10]; in particular, the *wandering exponent* ζ in the scaling relation $\ll (u(x,t)-u(0,t))^2 \gg \sim\ x^{2\zeta}$ is $\zeta = 2/3$. (The double brackets indicate averages over all thermodynamic states and disorder configurations.) This scaling relation, which describes the spatial distribution of the low-lying metastable states, is the starting point for the theory of creep-type dynamics for elastic manifolds developed in [5].

The dynamics of the driven string falls into two major regimes, depending on the magnitude of the driving force F. At very large driving forces, the string is in the *viscous-flow regime*, and the effect of the pinning centers is very weak. Perturbation theory with respect to weak disorder applies, and disorder results in a renormalization of the damping constant [11]. The deviation of the velocity from purely viscous flow, $\delta v = v - F$, scales as $\delta v \sim F^{-1/2}$. Pinning becomes more effective as the driving force decreases. At zero temperature, the string becomes pinned at some *critical force* F_c, and the average string velocity is zero for all $F < F_c$ [12]; in type-II superconductors, this critical force corresponds to the critical current. The quantity F_c has been estimated [5, 6], $F_c = c_f(\rho U_p^2)^{2/3}C^{-1/3}$, where c_f is a numerical factor. At finite temperatures, the transition from the unpinned to the pinned regime is blurred, and the string moves even at very small forces. The string spends most of its time in the low-lying metastable states induced by the disorder, and string motion occurs via a sequence of thermally activated elementary jumps of string segments into neighboring metastable states favored by the applied force. A temperature-dependent critical force $F_c(T)$ can be defined conceptually by the equality $|\delta v(F_c(T))| = v(F_c(T))$ [13]. According to [5, 6], $F_c(T)$ is slowly varying, $F_c(T) \approx F_c(0)$, for $0 < T < T_{dp}$, but $F_c(T)$ drops dramatically as T increases above T_{dp}. The characteristic *depinning temperature* T_{dp} is given by $T_{dp} = c_t(\rho U_p^2 C)^{1/3}$, where c_t is a numerical factor.

The motion of the string for $F \ll F_c(T)$ has been analyzed by Ioffe and Vinokur [5]. The activation barriers diverge as $U(F) \sim T_{dp}(F_c/F)^{\mu}$, and the string velocity depends strongly nonlinearly upon the driving force,

$$v = \begin{cases} v_0 \exp\left[-c_v(T_{dp}/T)(F_c/F)^{\mu}\right], & T < T_{dp}, \\ v_0 \exp\left[-c_v(F_c(T)/F)^{\mu}\right], & T > T_{dp}. \end{cases} \tag{4}$$

Here, c_v is a positive constant. Central to the analysis in [5] is the *assumption* that there is a unique scale for disorder-induced energy fluctuations. Energy barriers between metastable states then scale in the same manner as fluctuations in the free energy between the low-lying states, and one finds that $\mu = 1/4$. One of our objectives was to corroborate this assumption in some large-scale computational experiments.

In the computational experiments, we assumed that the string is generally oriented in the x direction and moves in the positive y direction. We used a regular square lattice of unit spacing with L mesh points in both directions. The lattice is seeded with identical Gaussian pinning potentials at randomly selected sites, with a prescribed density ρ; the seeded lattice is extended periodically in the x direction. The discretization of the thermal fluctuations was accomplished by means of a model developed by Schneider and Stoll [14]; the temperature was calibrated by simulating Brownian motion.

The following results were all obtained with the parameter values $C = 3.0$, $U_p = 0.3$, and $\rho = 0.3$. This choice puts us in the regime of weak pinning ($\rho C \gg U_p$). In most cases, we took $L = 1024$; for some experiments, we went as high as $L = 2048$. In general, extremely long run times are necessary for reliable statistics, especially in the regime of very small applied forces.

Figure 1 shows the variation of the average string velocity v with the driving force F for $T = 0.3$. One clearly distinguishes the viscous-flow regime (large F), a transition regime (intermediate F), and a strongly nonlinear regime (small F). In the viscous-flow regime, we find that $\delta v \sim F^{-1/2}$, as predicted by theory. In the transition regime, we observe a remarkable stepwise behavior of v. The steps reflect an avalanche-like motion of the string near the depinning transition. Large segments of the string are getting stuck in regions where the pinning forces roughly balance the driving force. As the driving force tends to a critical value, the size of the regions where the string is coherently pinned becomes comparable to the length of the string. Local thermal fluctuations then trigger jumps of large sections of the string toward regions of high defect densities, where the potential is strong enough to pin these segments. In simulations at zero temperature, Dong et al. [12] also observed that large segments of the string are pinned when the pinning forces roughly balance the driving force. As the driving force decreases further below a critical value, the string transits into the pinned (glassy) state, where its motion is determined by the thermally activated jumps of relatively small string segments between neighboring metastable states. For very small driving forces, theory [5] predicts a formula of the form (4). The choice $\mu = 1/4$ provides a very good fit for the data of Figure 1, see Figure 2. However, other values of the exponent (for example, $\mu = 1/3$) give an almost equally good fit, so the best we can say is that the results of our computational experiments are consistent with theoretical predictions. Unfortunately, the available data are insufficient to determine the constant c_v in the exponent in (4) or to verify the temperature dependence of the slope of the $\log v$ vs. $1/F^\mu$ graph for fixed μ.

From the data we infer that the preexponential factor v_0 in (4) varies linearly with F. We can therefore cross check the value of the exponent μ by plotting $\log(|\log(v/F)|)$ vs. $\log F$; Figure 3 shows the result, again for $T = 0.3$. The linear part at small forces corresponds to glassy motion. One observes a sharp transition, separating the (linear) glassy regime from a viscous regime at large forces. From a linear least-squares fit we find the slope of the linear part, $\mu = 0.248 \pm 0.016$, in excellent agreement with the predicted value, $\mu = 1/4$. Nevertheless, we do not consider this result as conclusive to confirm the validity of (4). The value of μ was found on the basis of a rather narrow range of F (about one decade). Furthermore, none of the computations penetrated really deeply into the glassy regime ($F \ll F_c$).

We identify the value of F indicated by the vertical line in Figure 3, where the system transits from glassy to viscous behavior, with the critical force $F_c(T)$. Its variation with temperature is shown in Figure 4. Note the low-temperature region, where F_c is almost independent of the temperature: $F_c(T) = 0.101 \pm 0.003$, in good agreement with the value 0.105 ± 0.003 found from independent simulations at $T = 0$. We also find a sharply defined value of the depinning temperature, where F_c begins to drop rapidly: $T_{dp} = 0.068 \pm 0.008$. The least-squares fit of the formula $F_c(T) \simeq T^{-\alpha}$ to the data of Figure 4 in the region where F_c varies strongly with T gives $\alpha = 0.095 \pm 0.003$. This value disagrees with the predicted value $\alpha = 7$ [5] .

Given the analytical expressions $F_c(0) = c_f(\rho U_p^2)^{2/3} C^{-1/3}$ and $T_{dp} = c_t(\rho C U_p^2)^{1/3}$ obtained in [5], we deduce the following values for the constants c_f and c_t from our numerical simulations: $c_f \approx 1.6 \pm 0.2$ and $c_t \approx 0.16 \pm 0.02$.

The *dynamical roughening* is measured by $w(l) = < (u(l,t) - u(0,t))^2 >^{1/2}$. Assuming that $w(l) \sim l^\zeta$, we find $\zeta \approx 0.7 \pm 0.05$ below and near the transition regime, at least at intermediate length scales ($l \approx 50 - 100$). This result is close to the static exponent $\zeta_s = 2/3$ and confirms the qualitative understanding of glassy motion as a sequence of rare jumps between static metastable states. At large length scales, Horvath et al. have reported a crossover to $\zeta = 0.5$ in experiments on the interfaces between two liquids in a porous medium [15]. We find that, for $F \gg F_c$, ζ crosses over to a smaller value $\zeta \approx 0.5 \pm 0.05$, which is characteristic for the roughening induced by thermal noise. This result is in agreement with [7], where the KPZ model was used to find the fast viscous motion of the string, and with previous results for $T = 0$ [12]. In the unpinned region, quenched disorder is effectively reduced to thermal noise [7].

In summary: (i) The elastic string shows glassy behavior at small driving forces. Our data for the average string velocity are consistent with (4), where μ is close to the predicted value $\mu = 1/4$. The critical force F_c varies with temperature like $T^{-\alpha}$, where $\alpha = 0.095 \pm 0.003$. (ii) The results for $F \gg F_c$ show excellent agreement with the predictions of perturbation theory. (iii) In the transition region between the viscous and thermally activated regimes, the string motion is governed by avalanche-like processes. (iv) The roughening of the string agrees with the qualitative picture of glassy motion if $F \ll F_c$ and with the asymptotic results for the KPZ model if $F \gg F_c$. (v) Although the observed dependence of v on F is consistent with the predictions of the theory of nucleation motion, the data are insufficient to conclude that there is a unique scale of disorder-induced energy fluctuations—the fundamental assumption underlying the derivation of (4).

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FIGURE CAPTIONS[1]

Figure 1. Average string velocity v against the driving force F at $T = 0.3$.

Figure 2. Linear variation of $\log v$ with $(1/F)^{\mu}$ for $T = 0.3$. The solid line represents a least-squares fit for $\mu = 1/4$; the r.m.s. error of the data (marked by $\times$) is 0.018. The value $(1/F_c(0))^{1/4} = 1.774$ is beyond the left margin.

Figure 3. Demonstration of glassy dynamics; $\log(|\log(v/F)|)$ against $\log F$ for $T = 0.3$. The slope of the dashed line is -0.248. The vertical line indicates the position of $F_c(0.3)$.

Figure 4. Temperature dependence of the critical force F_c.

[1] Figures are not available electronically. Contact authors for figures.