The Problem of Strategy and Hyperresolution

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Abstract. This article is the thirty-first of a series of articles discussing various open research problems in automated reasoning. The problem proposed for research asks one to find a strategy that can be coupled with the inference rule hyperresolution to control the behavior of an automated reasoning program as effectively as does paramodulation.

Key words. Automated reasoning, hyperresolution, paramodulation, strategy, unsolved research problem.

Question: *On problems such as the commutator theorem of group theory, does there exist a strategy that enables a reasoning program to employ hyperresolution with the same effectiveness as paramodulation?*

(This question is the fourth of 33 problems proposed for research in [4] and will be referred to as Research Problem 4 throughout this article. All references to sections, chapters, test problems, and such also refer to [4].)

One of the benchmark problems in automated reasoning is the commutator theorem (Test Problem 2, Section 6.1.2) taken from group theory. The hypothesis of the theorem states that the cube of every x is the identity e. With that hypothesis, one can prove that $[[x,y],y] = e$ for all x and y, where $[x,y]$ is the product of x, y, x inverse, and y inverse. A proof has been obtained in slightly more than 2 seconds of CPU time on an IBM 370/195 with the automated reasoning program AURA (AUtomated Reasoning Assistant) [3] employing the inference rule paramodulation [1, 6, 7]. With the same computer and program, but with the inference rule hyperresolution [2] replacing paramodulation, a proof was obtained in approximately 100 seconds of CPU time. We note that AURA, written in IBM assembly language, is now impossible to obtain; therefore, we recommend [5], which contains a diskette of the powerful reasoning program OTTER. Also given in [5] is a detailed treatment of both paramodulation and hyperresolution. In Research Problem 4, the object is to find a strategy that controls hyperresolution in such a manner that its behavior essentially matches that of paramodulation.

To test a proposed solution, one can focus on problems somewhat reminiscent of the commutator theorem. The theorem that states that Boolean rings are commutative (Test Problem 8, Section 6.2.1) is a good example. Various theorems from Boolean algebra are also good choices. Regardless of the choice, for each problem, the results should be compared when one employs hyperresolution and a notation that avoids, where possible, the use of equality with the results obtained when one employs paramodulation and a notation that emphasizes the use of equality.

To attack Research Problem 4, one might consider some area of mathematics, such as group theory, carefully compare the proofs that are obtained from using hyperresolution with those obtained from using paramodulation, and then analyze the results. In particular, one might use the axioms (GT-

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1) through (GT-17) with hyperresolution as the inference rule, and then use the axioms (GTEQ-1) through (GTEQ-6) with paramodulation as the inference rule (see Section 6.1 for both clause sets). The attack might proceed by developing the following notion, for which we can provide only a few hints.

One can study the relation between the two approaches by mapping unit clauses of the form P(r,s,t) to unit clauses of the form EQUAL(prod(r,s),t). The image of a nonunit clause under this mapping can be obtained by first applying hyperresolution to the nonunit clause as nucleus and (some number of copies of) the clause (GT-7), the axiom for closure, as satellite. For example, both clause (GT-5) and clause (GT-6) are mapped to clause (GTEQ-5)—associativity in one notation is mapped to associativity in the other. One must, of course, decide on an ordering of the arguments in equality clauses, as well as managing other such details. No doubt, an understanding of the role of demodulation in the two approaches will be needed.

The analysis is further complicated by the fact that the clause for closure (GT-7) is mapped to an instance of reflexivity (GTEQ-6), as discussed in Section 5.9. Even if the reasoning program finds distinctly different proofs for the same theorem, when using hyperresolution versus using paramodulation, one can, nevertheless, use the two computer proofs and the given mapping to produce by hand the missing and equivalent proofs. Finally, one can study the possible reasons that might explain why the program fails to find equivalent proofs.

Since paramodulation has proved very effective for certain classes of problem in which equality plays an important role, a solution to Research Problem 4 might sharply increase the effectiveness of hyperresolution for problems in which equality plays little or no role.

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