Basic Research Problems:

The Problem of Hyperparamodulation and Nuclei*

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Abstract. This article is the thirty-third of a series of articles discussing various open research problems in automated reasoning. The problem for research asks one to establish criteria for allowing certain—but not all—new clauses to become nuclei when using the inference rule hyperparamodulation or hyperresolution.

Key words: Automated reasoning, hyperparamodulation, hyperresolution, nuclei, unsolved research problem.

Question: *What criteria should be used to cause a reasoning program to adjoin new nuclei when using the inference rule of hyperparamodulation or of hyperresolution?*

(This question is the tenth of 33 problems proposed for research in [1] and will be referred to as Research Problem 10 throughout this article. All references to sections, chapters, test problems, and such also refer to [1].)

The inference rule of hyperparamodulation (see Section 5.9 for a discussion of this inference rule), as it is currently described and implemented [2], permits the user to designate from among the input clauses those clauses that are not to be used as satellites. With this option, the user can, for example, prevent the unit clause for associativity from being used as a satellite. If this action is not taken, experiments show that the reasoning program can drown in unwanted information. On the other hand, all input clauses are allowed to be nuclei. (For a detailed discussion and examples of hyperresolution, satellites, and nuclei, see [3].) All clauses that are deduced and added to the database of information are also allowed to be nuclei. Unfortunately, no criteria are currently used to decide which new clauses should be excluded from being nuclei.

Research Problem 10 focuses on a variant of hyperparamodulation that would allow additional nuclei to be adjoined during the actual run, but would apply some set of criteria to prevent all new clauses from becoming nuclei. Specifically, Research Problem 10 asks for criteria for choosing which nuclei to adjoin during the run with the object of increasing the effectiveness of the reasoning program. A related question focuses on which clauses, input or deduced, should be treated as satellites. An additional question focuses on which terms in the nuclei should be accepted as *into* terms.

Let us briefly consider some examples. One might conjecture that the clause

EQUAL(inv(inv(x)),x)

whether input or deduced, should be classed as a satellite, but not as a nucleus. On the other hand, perhaps the clause

EQUAL(inv(prod(x,y)),prod(inv(y),inv(x)))

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whether input or deduced, should be classed as both satellite and nucleus. For a third example, if the program deduces a complex polynomial expression, one might reasonably conjecture that the new clause should be classed as a nucleus only. As for terms, one might begin by excluding variables as possible *into* terms. Then, one might require that either but not both arguments of a nucleus be admissible as *into* terms. In other words, one might decide to closely imitate hyperresolution; when hyperparamodulation is being applied to some nucleus which is itself a positive equality unit clause, and when a substitution has already occurred in one of the two arguments of the nucleus, then the other argument could not be used as an *into* term. If this restriction were the choice, then the clause (P-4) of Section 5.9 (see the Basic Research Problems article in the preceding issue of the *Journal of Automated Reasoning*) would not be deducible in one application of hyperparamodulation.

If a solution to Research Problem 10 could be found, perhaps the same or similar criteria could be employed, when using the inference rule hyperresolution, to adjoin new nuclei. Of course, since hyperresolution generates clauses with positive literals only, we have in mind some source other than hyperresolvents for new nuclei, possibly coupled with a relaxation of the current definition of hyperresolution. In particular, with regard to adjoining (after the program has begun its attack on a problem) new nuclei to be used by hyperresolution, a reasoning program's effectiveness might be increased if certain types of **if-then** statements were adjoined as new nuclei, and the criteria for inference rule application broadened. One might, for example, during the run, replace the syntactic criteria now used by some semantic criteria.

By adjoining new nuclei, the effectiveness might be increased measurably, for the reasoning program might then be able to avoid repeating long chains of inference that require considering a number of nuclei from among the input clauses. Alternatively, one might allow the reasoning program using hyperresolution to adjoin as new nuclei clauses that are obtained from newly deduced unit clauses. These new nuclei would be obtained by transforming the designated unit clause into nonunit clauses by using the inverse of a mapping of the type discussed in Section 5.9. For example, the approach under discussion would, if the program deduced the equality form of associativity, produce the two clauses (GT-5) and (GT-6) for associativity given in Section 5.9. For a second example, in the commutator theorem (Test Problem 2, Section 6.1.2), a reasoning program can deduce an equality unit clause that says that (the right associated form of) $xyxyxy = e$. This unit clause would be transformed into the clause

$$\neg P(x,y,z) \mid \neg P(y,z,u) \mid \neg P(x,u,v) \mid \neg P(y,v,w)\ P(x,w,e)$$

with four negative literals and one positive, which would then be used as a nucleus without modifying or extending the current definition of hyperresolution. Since both hyperparamodulation and hyperresolution afford a reasoning program the opportunity of taking larger steps, knowing which new nuclei to profitably adjoin during the run (to add to those from among the input clauses) might prove very significant.

To test a proposed solution to Research Problem 10, one might begin (using a notation that emphasizes the role of equality) with the theorem that states that Boolean rings are commutative (Test Problem 8, Section 6.2.1)—rings in which the square of x is x itself. If the hypothesis of this theorem is replaced by assuming the cube of x equals x, one can again prove that the ring is commutative (Test Problem 9, Section 6.2.2); since this theorem is substantially more difficult than the theorem about Boolean rings, this theorem provides a far better test of a proposed solution to Research Problem 10. If one is testing a solution to this problem that focuses on adjoining nuclei when using hyperresolution, one can use as tests of a proposed solution the same two theorems. However, one would then represent the two theorems by using a notation that avoids where possible the use of equality (see Section 6.2), especially if one is studying the alternative that requires broadening the definition of hyperresolution. The commutator theorem of group theory, phrased in the appropriate notation, also provides a good test for evaluating a proposed solution to either aspect of Research Problem 10.

References

1. Wos, L., *Automated Reasoning: 33 Basic Research Problems*, Prentice-Hall, Englewood Cliffs, New Jersey (1988).

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3. Wos, L., Overbeek, R., Lusk, E., and Boyle, J., *Automated Reasoning: Introduction and Applications*, 2nd ed., McGraw-Hill, New York (1992).