Dynamic Vortex Phases in Superconductors with Correlated Disorder

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The nature of driven motion of a vortex solid in the presence of a planar pinning defect is investigated by large-scale simulations based on the time-dependent Ginzburg-Landau equations. Three dynamic phases are identified and characterized by their relative positional and velocity correlations.

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Vortices in superconductors are a well-defined system of elastic lines or points interacting by electromagnetic and hydrodynamic forces, which can be driven through a field of pinning sites by an applied current. The dynamic response of the vortices controls most transport properties of superconductors and displays remarkable variety, including nonlinear effects, steady-state and avalanche dynamics, and thresholds for the onset of motion. A basic understanding of these dynamics is of fundamental interest and is an essential step in controlling vortex motion in technical applications.

Vortex motion may be classified generally as elastic or plastic. In elastic motion, each vortex keeps the same neighbors, while in plastic motion the neighbors change. Plastic vortex flow has been identified in molecular dynamics simulations [1] and in pioneering transport experiments on $NbSe_2$ [2]. Plastic-to-elastic dynamic transitions have been predicted analytically [3] and explored in neutron scattering experiments [4,5] and transport measurements [6,7]. In this paper, we use large-scale simulations of the time-dependent Ginzburg-Landau equations to investigate the internal structure of elastic and plastic motion, following the individual motions of hundreds of vortices in the presence of a controlled driving force and a planar pinning defect. We find two distinct plastic phases and an elastic phase, each with different internal symmetry. We identify and compare the characteristic features of each dynamic phase and the physical conditions favoring each.

The simulations are based on the time-dependent Ginzburg-Landau equations [8,9],

$$\frac{\hbar^2}{2m_s D}\frac{\partial\psi}{\partial t} = -\frac{\delta\mathcal{L}}{\delta\psi^*}, \quad \frac{\sigma}{c^2}\frac{\partial\mathbf{A}}{\partial t} = -\frac{\delta\mathcal{L}}{\delta\mathbf{A}} - \frac{1}{4\pi}\nabla\times\nabla\times\mathbf{A},$$

$$\mathcal{L} = a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{2m_s}\left|\left(\frac{\hbar}{i}\nabla - \frac{e_s}{c}\mathbf{A}\right)\psi\right|^2,$$

where ψ is the complex order parameter, $\mathbf{A}$ the vector potential, and $\mathcal{L}$ the Helmholtz free-energy density; the other symbols have their usual meaning. Computational details have been described in [10].

The simulated sample was a rectangular cylinder, infinitely long and homogeneous in the field direction. The sample had two parallel free surfaces defined by the boundary condition $\mathbf{J}_s \cdot \mathbf{n} = 0$ ($\mathbf{J}_s$ is the supercurrent density). A transport current was induced by a field differential between the free surfaces. Periodic boundary conditions were applied along the current direction to avoid edge effects. The sample measured 32λ (between the free surfaces) by 48λ (in the current direction), where λ is the magnetic penetration depth. The computational grid consisted of 256×384 points, which gave two grid points per coherence length ξ when the Ginzburg-Landau parameter $\kappa = 4$ ($\kappa = \lambda/\xi$). The absence of thermal fluctuations restricts our results to the vortex solid.

Correlated disorder was introduced within a slab of thickness 2ξ oriented parallel to the field direction and $45°$ to the free surfaces, a geometry common for twin boundaries in YBCO. The disorder was modeled by a reduction of the condensation energy to a mean value of 56% of the bulk value, accompanied by Gaussian spatial fluctuations with a standard deviation of 25% of the bulk value. The fluctuations in condensation energy provided resistance to vortex motion within the twin boundary, as observed experimentally [11]. There was no disorder in the bulk of the sample outside the slab.

Simulations were run for three transport currents representing approximately 2% (*weak current*), 4% (*intermediate current*), and 8% (*strong current*) of the depairing current. The calculated quantities ψ and $\mathbf{A}$ on the computational grid at each time step provided a record of the evolution of the system. Each simulation required approximately 100 hours on 16 processors of the IBM Scalable POWERparallel (SP) computational system at Argonne National Laboratory. We report only the behavior in the steady-state portion of the time window, which was characterized by the requirement that the number of vortices in the sample vary by less than 1%.

Figure 1(a) shows the Delaunay triangulation of the positions of the vortices at weak current for a typical time step. There is strong spatial correlation among the moving vortices. The vortices accommodate the twin boundary by making its orientation a close-packed direction. This imposed orientation extends over a long range, up to the dimensions of the simulated sample. However, it competes with another orientation imposed by the free surfaces, which are also energetically favorable close-packed directions [12]. The 15° misorientation between these two directions produces some of the defects in the vortex lattice, indicated by open and filled circles in Fig. 1. Additional defects in the form of dislocations appear on the twin boundary to accommodate the incommensurability of the vortex densities there and in the bulk.

Figure 2(a) shows the vortex trajectories at weak current. The motion of the vortices is strongly influenced by the surface barrier arising from the boundary condition $\mathbf{J_s} \cdot \mathbf{n} = 0$. For a homogeneous boundary, the critical Lorentz force needed to overcome the surface barrier has been calculated analytically and confirmed in simulations [13]. The twin boundary perturbs the surface barrier locally where the two meet, creating a favorable entry point for vortices.

The vortex motion at weak current is plastic, as indicated by the irregular velocity profile over the sample. A comparison of Figs. 1(a) and 2(a) shows that the motion is correlated with the location of the defects. Detailed examination of successive time steps reveals that vortex motion at the twin boundary occurs via sliding of dislocations. There is significant correlation in the velocities, with a correlation length extending up to approximately four vortex spacings above the twin boundary. Correlated regions are separated by discontinuities in the *magnitude* and *direction* of the velocity. While magnitude discontinuities might be expected, as the vortices slide past each other, direction discontinuities as large as 60° (which occur at the intersection of the twin boundary and the left free boundary) are surprising at first sight. At weak current, the pinning forces dominate the driving Lorentz force, and the vortices on the twin boundary are immobilized. The boundary presents an impenetrable barrier to vortex motion. The vortices respond by moving predominantly parallel to the twin boundary, even at distant points [14].

A key feature of the plastic motion at weak current is revealed in Fig. 2(a): The velocity directions are primarily along the close-packed directions. The latter define the sliding planes for dislocations. This restriction has a significant implication for the symmetry of the dynamic state: *Orientational order is preserved.* This feature is examined quantitatively below.

The principle of motion along close-packed directions explains the velocity direction discontinuities in Fig. 2(a). In the upper part of the sample, the twin boundary blocks two of the three close-packed directions, leaving only the parallel direction available for motion. Because orientational order is preserved over long distances, the twin boundary guidance operates throughout the sample. Just below the left end of the twin boundary, the barrier effect is absent, and all three close-packed directions are available for motion. The vortices choose to move along the direction closest to the driving Lorentz force. Thus, the velocity direction discontinuity is an exchange of one close-packed direction for a more favorable one.

Figure 2(b) shows two important changes in the character of the vortex motion at intermediate current. First, near the twin boundary, the trajectories are much more disorganized than at weak current, and neither the restriction of motion to a few directions nor the correlation between neighboring velocities can be seen. The disorder in the vortex trajectories is matched by disorder in the vortex positions shown in Fig. 1(b). Near the twin boundary, there are many defects, and no universal close-packed direction can be identified. This uncorrelated plastic motion is in sharp contrast to the correlated plastic motion at weak current.

The physical origin of uncorrelated plastic motion is the breakdown of the extended nature of the twin boundary. At intermediate current, the boundary is penetrated at random positions along its length, depending on the relative strength of the Lorentz force and the local pinning force. This random penetration precludes the establishment of a universal close-packed direction for the vortices and upsets the associated induced orientation and velocity correlation.

The second change at intermediate current is the transition from plastic to elastic motion, which has occurred far from the twin boundary. The lattice structure shown in Fig. 1(b) is nearly perfect there. Pinning is absent, and the Lorentz force is dominant. The vortices move nearly uniformly in the direction of the driving Lorentz force at approximately equal velocities.

At strong current, the Lorentz force overwhelms the twin boundary pinning, and the motion is elastic everywhere. Figure 2(c) shows that most trajectories suffer only a slight perturbation at the twin boundary.

The orientational order in the dynamic states is examined in Fig. 3, which shows the angular distribution of the bonds connecting neighboring vortices, and in Table 1, which shows the hexatic bond-orientational order parameter $<\mathrm{e}^{i6\theta}>$ (θ is the bond angle). All bonds between neighboring vortices at the last time step in the simulation have been included. For reference, Table 1 also shows the order parameter for simulations on the same sample without a twin boundary. Data for a region centered on the twin boundary, extending 5.625λ (approximately three lattice spacings at intermediate current) on either side, and for the complementary region are shown separately. At weak current and near the twin boundary, the six narrow peaks centered on $45° + n60°$ show hexatic symmetry aligned with the twin boundary. Away from the twin boundary, competing peaks at $30° + n60°$ reflect the influence of the sample edges in establishing the close-packed directions. The order parameter reflects this angular shift in its phase, which gives the orientation of the hexatic pattern if the order parameter is large. The magnitude of the order parameter in both regions at weak current indicates significant hexatic order, somewhat reduced from the reference simulation without twin boundary where the hexatic orientation is determined only by the sample edges. At intermediate current and near the twin boundary, the hexatic order is severely reduced. Away from the twin boundary, hexatic order associated with elastic motion sets in, and the hexatic pattern aligns with the Lorentz force. The elastic flow is not fully developed, because of interference by the twin boundary. The reference simulation shows the order parameter for fully developed elastic flow. At strong current, the magnitude of the hexatic order parameter is reduced in both the twinned and untwinned simulations, because of the appearance of a defect superstructure in the moving lattice induced by the field gradient of the transport current [13]. This effect obscures the relatively perfect elastic flow expected for large driving forces.

The simulations reported here have identified three dynamic phases of a vortex solid driven by a Lorentz force in the presence of correlated disorder: (i) elastic flow, (ii) correlated plastic flow, and (iii) uncorrelated plastic flow. Elastic flow is characterized by long-range correlation in the positions and velocities of the vortices and by the presence of translational periodicity and hexatic orientational order in the moving vortex lattice. It occurs when the driving Lorentz force is dominant, causing the vortex velocities to align with its direction. Plastic flow occurs when the pinning forces compete effectively with the Lorentz force and the vortex interactions. The vortex velocities generally do not align with the Lorentz force. In correlated plastic flow, the pinning forces in the planar defect dominate the Lorentz force, creating an impenetrable barrier for vortex motion. The extended nature of the planar defect is manifest, and the vortex system displays hexatic symmetry commensurate with the orientation of the defect plane. There is significant correlation in the vortex velocities, though the correlation length does not extend over the whole sample; rather, correlated regions are separated by discontinuities in the direction and magnitude of the velocity. Correlated plastic flow is mediated by weakly interacting dislocations sliding along the slip planes. In uncorrelated plastic flow, both the velocity correlation length and the positional correlation length are shorter than the intervortex spacing. There is neither translational periodicity nor significant hexatic orientational symmetry. Uncorrelated plastic flow has its physical origin in large spatial fluctuations of the relative strength of the Lorentz and pinning forces. These fluctuations are a general feature of depinning transitions and appear when the Lorentz force is nearly balanced by the pinning forces. Uncorrelated plastic flow is also expected for motion through a field of random disorder, where the fluctuations arise from the random positioning of the pin sites, rather than from the strength of the pins. The identification of symmetry in the plastic and elastic dynamic response of the vortices enables the distinction of well-defined dynamic phases and provides a means for characterizing dynamic phase transitions in moving vortex systems.

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Fig. 1. Delaunay triangulation of the vortex positions at one time step of the steady-state motion. Solid lines represent bonds between nearest neighbors. Open circles: vortices with five neighbors; filled circles: vortices with seven neighbors. (a) Weak current. (b) Intermediate current.

Fig. 2. Vortex trajectories during the steady-state motion. (a) Weak current. (b) Intermediate current. (c) Strong current. The Lorentz force acts to the right.

Fig. 3. Distribution of bond angles for all pairs of neighbors at the last time step during the steady-state motion.

Table 1. Normalized magnitude (u) and phase (ϕ) of the hexatic bond-orientational order parameter, $<e^{i6\theta}> \equiv ue^{i\phi}$; ϕ has been divided by 6 and taken modulo 60°, to give a bond direction in the first sextant.

	Twinned				Untwinned	
	Near Bndry		In Bulk			
Current	u	ϕ	u	ϕ	u	ϕ
Weak	0.631	44.2°	0.608	35.4°	0.834	28.9°
Interm.	0.264	13.0°	0.548	2.45°	0.787	3.06°
Strong	0.513	10.2°	0.596	11.7°	0.567	4.53°