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Short Equational Bases for Ortholattices: Proofs and Countermodels

by

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Contents

Abstract **1**

1 Introduction **1**

2 Equational Bases **1**

2.1 In Terms of Join/Meet/Complement . . . 1

2.1.1 Lattice Theory . . . 2

2.1.2 Ortholattices . . . 3

2.1.3 Orthomodular Lattices . . . 5

2.1.4 Modular Ortholattices . . . 7

2.1.5 Boolean Algebra . . . 11

2.2 In Terms of the Sheffer Stroke . . . 33

2.2.1 Ortholattices in Terms of the Sheffer Stroke . . . 34

2.2.2 Orthomodular Lattices in Terms of the Sheffer Stroke . . . 37

2.2.3 Modular Ortholattices in Terms of the Sheffer Stroke . . . 41

2.2.4 Boolean Algebra in Terms of the Sheffer Stroke . . . 45

2.3 Finding and Proving the Multiequation Bases . . . 50

2.4 Are There Simpler Multiequation Bases? . . . 50

3 Single Axioms **50**

3.1 Generating and Filtering Candidates . . . 51

3.2 Finite Ortholattices . . . 51

3.3 Collecting and Applying Filters . . . 51

3.4 Trying to Prove That Candidates Are Single Axioms . . . 63

3.5 Single Axioms for $\mathcal{OL}$, $\mathcal{OML}$, and $\mathcal{MOL}$ . . . 63

3.5.1 A Single Axiom for Ortholattices . . . 64

3.5.2 A Single Axiom for Orthomodular Lattices . . . 69

3.5.3 A Single Axiom for Modular Ortholattices . . . 73

4 The Computer Programs **82**

5 Conclusion **82**

References **82**

Short Equational Bases for Ortholattices: Proofs and Countermodels

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Abstract

This document contains proofs and countermodels in support of the paper "Short Equational Bases for Ortholattices", by the same set of authors. In that paper, short single axioms for ortholattices, orthomodular lattices, and modular ortholattices are presented, all in terms of the Sheffer stroke. The ortholattice axiom is the shortest possible. Other equational bases in terms of the Sheffer stroke and in terms of join, meet, and complement are presented. Computers were used extensively to find candidates, reject candidates, and search for proofs that candidates are single axioms.

1 Introduction

This report supports the (primary) paper "Short Equational Bases for Ortholattices" and the Web page *Short Equational Bases for Ortholattices: Web Support* [9]. The primary paper is essentially an extended abstract intended for a wide audience, the Web page is intended for immediate support, and this report is intended for archival support.

The overall structure of the three documents is similar, with the same section titles (even when the sections are empty). The primary paper contains no proofs, countermodels, or filters. The Web page contains proofs, countermodels, filters, and information on the software and methods used. This report contain the proofs, countermodels, and filters.

2 Equational Bases

First we define a chain of varieties from lattices to Boolean algebra in terms of join, meet, and complement. Then we go from ortholattices to Boolean algebra in terms of the Sheffer stroke.

2.1 In Terms of Join/Meet/Complement

This section contains multiequation bases, in terms of join, meet, and complement, for lattices ($\mathcal{L}$), ortholattices ($\mathcal{OL}$), orthomodular lattices ($\mathcal{OML}$), modular ortholattices ($\mathcal{MOL}$), and Boolean algebra ($\mathcal{BA}$). This development is not far from a standard development; and in the cases where it does depart, we prove the corresponding theorems and other theorems that may be of interest. For example, our basis for lattice theory does not contain commutativity, so we prove it; and we use a nonstandard modularity axiom, so we prove it equivalent (given

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lattice theory) to the standard axiom. Note that each of the bases for $\mathcal{OL}$, $\mathcal{OML}$, $\mathcal{MOL}$, and $\mathcal{BA}$ contains the equation $x \wedge y = c(c(x) \vee c(y))$, which allows us, if we wish, to rewrite the bases to be in terms of join and complement only.

Also, with the exception of $\mathcal{BA}$, we prove independence of the bases by giving countermodels found by Mace2. Many of the independence proofs are trivial, and we could give arguments without the help of Mace2, but we have included the Mace2 countermodels to emphasize the "automatic" nature of the work.

2.1.1 Lattice Theory

The following is a 4-basis for lattice theory ($\mathcal{L}$) in terms of join ($\vee$) and meet ($\wedge$).

$x \vee (y \vee z) = y \vee (x \vee z)$	% AJ
$x \wedge (y \wedge z) = y \wedge (x \wedge z)$	% AM
$x \vee (x \wedge y) = x$	% B1
$x \wedge (x \vee y) = x$	% B2

Note that this basis is self-dual; that is, the dual of each member is also a member.

A more standard basis for lattice theory is the 6-basis (also self-dual) consisting of B1, B2, and the commutativity and associativity of meet and join. Deriving commutativity of join (or of meet) from the 4-basis above shows that it is equivalent to the 6-basis. The following is an Otter proof of commutativity (idempotence is also proved).

Proof LT1

1	$A \vee B \neq B \vee A \mid \$Ans(CJ)$	[]
2	$A \vee A \neq A \mid \$Ans(IJ)$	[]
4	$x \vee (y \vee z) = y \vee (x \vee z)$	[]
5	$x \wedge (y \wedge z) = y \wedge (x \wedge z)$	[]
6	$x \vee (x \wedge y) = x$	[]
8	$x \wedge (x \vee y) = x$	[]
12	$x \vee (y \vee (x \wedge z)) = y \vee x$	[6 $\rightarrow$ 4]
14	$x \wedge x = x$	[6 $\rightarrow$ 8]
18	$x \vee x = x$	[8 $\rightarrow$ 6]
20	$\$Ans(IJ)$	[18,2]
21	$x \wedge (y \wedge x) = y \wedge x$	[14 $\rightarrow$ 5]
74	$x \vee (y \wedge x) = x$	[21 $\rightarrow$ 6]
169	$x \vee y = y \vee x$	[74 $\rightarrow$ 12]
170	$\$Ans(CJ)$	[169,1]

The following two Mace2 jobs show independence of the $\mathcal{L}$ 4-basis {AJ, AM, B1, B2}.

Countermodel LTa

```
x ^ (y ^ z) = y ^ (x ^ z).     % AM
x v (x ^ y) = x.               % B1
x ^ (x v y) = x.               % B2
A v (B v C) != B v (A v C).    % denial of AJ
```

C: 0 B: 0 A: 1

$\wedge$:	0	1
0	0	0
1	0	1

$\vee$:	0	1
0	0	0
1	1	1

Table 1: LTa.out

Countermodel LTb

```
x v (y v z) = y v (x v z).     % AJ
x ^ (y ^ z) = y ^ (x ^ z).     % AM
x ^ (x v y) = x.               % B2
A v (A ^ B) != A.              % denial of B1
```

B: 0 A: 1

$\vee$:	0	1
0	0	0
1	0	0

$\wedge$:	0	1
0	0	1
1	1	0

Table 2: LTb.out

2.1.2 Ortholattices

The following is a 5-basis for ortholattices ($\mathcal{OL}$) in terms of join ($\vee$), meet ($\wedge$), and complement (c).

$x \vee (y \vee z) = y \vee (x \vee z)$	% AJ
$x \vee (x \wedge y) = x$	% B1
$x \wedge y = c(c(x) \vee c(y))$	% DM
$c(c(x)) = x$	% CC
$x \vee c(x) = y \vee c(y)$	% ONE

If we start with the lattice theory 4-basis {AJ, AM, B1, B2} and restrict that to the ortholattices by adding {DM, CC, ONE}, then AM and B2 become dependent. The following are Otter proofs.

Proof OL1

1	$x = x$	[]
2	$x \vee (y \vee z) = y \vee (x \vee z)$	[]
6,5	$x \wedge y = c(c(x) \vee c(y))$	[]
8,7	$c(c(x)) = x$	[]
10	$A \wedge (B \wedge C) \neq B \wedge (A \wedge C) \mid \$Ans(AM)$	[]
11	$c(c(B) \vee (c(A) \vee c(C))) \neq c(c(A) \vee (c(B) \vee c(C))) \mid \$Ans(AM)$	[10,6,6,8,6,6,8]
45	$c(c(A) \vee (c(B) \vee c(C))) \neq c(c(A) \vee (c(B) \vee c(C))) \mid \$Ans(AM)$	[2 $\rightarrow$ 11]
46	$\$Ans(AM)$	[45,1]

Proof OL2

1	$x = x$	[]
3	$x \vee (x \wedge y) = x$	[]
6,5	$x \wedge y = c(c(x) \vee c(y))$	[]
8,7	$c(c(x)) = x$	[]
10	$A \wedge (A \vee B) \neq A \mid \$Ans(B2)$	[]

11	$c(c(A) \vee c(A \vee B)) \neq A \mid \$Ans(B2)$	[10,6]
12	$x \vee c(c(x) \vee c(y)) = x$	[3:6]
33	$x \vee c(c(x) \vee y) = x$	[7 → 12]
50,49	$c(x) \vee c(x \vee y) = c(x)$	[7 → 33]
53	$A \neq A \mid \$Ans(B2)$	[11:50,8]
54	$\$Ans(B2)$	[53,1]

The following Mace2 jobs show that the $\mathcal{OL}$ 5-basis {AJ, B1, DM, CC, ONE} is independent.

Countermodel OLa

```
x v (x ^ y) = x.              % B1
x ^ y = c(c(x) v c(y)).       % DM
c(c(x)) = x.                  % CC
x v c(x) = y v c(y).          % ONE
A v (B v C) != B v (A v C).   % denial of AJ
```

C: 0 B: 0 A: 1

c:	0	1	2
	0	2	1

∧:	0	1	2
0	0	0	0
1	1	1	0
2	2	0	2

∨:	0	1	2
0	0	0	0
1	1	1	0
2	2	0	2

Table 3: OLa.out

Countermodel OLb

```
x v (y v z) = y v (x v z).    % AJ
x ^ y = c(c(x) v c(y)).       % DM
c(c(x)) = x.                  % CC
x v c(x) = y v c(y).          % ONE
A v (A ^ B) != A.             % denial of B1
```

B: 0 A: 1

c:	0	1
	0	1

∨:	0	1
0	0	0
1	0	0

∧:	0	1
0	0	0
1	0	0

Table 4: OLb.out

Countermodel OLc

```
x v (y v z) = y v (x v z).    % AJ
x v (x ^ y) = x.              % B1
c(c(x)) = x.                  % CC
x v c(x) = y v c(y).          % ONE
A ^ B != c(c(A) v c(B)).      % denial of DM
```

B: 0 A: 0

c:	0	1
	1	0

∨:	0	1
0	0	0
1	0	1

∧:	0	1
0	1	0
1	1	1

Table 5: OLc.out

Countermodel OLd

```
x v (y v z) = y v (x v z).    % AJ
x v (x ^ y) = x.              % B1
x ^ y = c(c(x) v c(y)).       % DM
x v c(x) = y v c(y).          % ONE
c(c(A)) != A.                 % denial of CC
```

A: 0

c:	0	1	2
	1	2	1

$\vee$:	0	1	2
0	0	1	0
1	1	1	1
2	0	1	2

$\wedge$:	0	1	2
0	2	2	2
1	2	1	2
2	2	2	2

Table 6: OLd.out

Countermodel OLe

```
x v (y v z) = y v (x v z).    % AJ
x v (x ^ y) = x.              % B1
x ^ y = c(c(x) v c(y)).       % DM
c(c(x)) = x.                  % CC
A v c(A) != B v c(B).         % denial of ONE
```

B: 0 A: 1

c:	0	1	2
	2	1	0

$\vee$:	0	1	2
0	0	0	0
1	0	1	1
2	0	1	2

$\wedge$:	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

Table 7: OLe.out

2.1.3 Orthomodular Lattices

The following is a 4-basis for orthomodular lattices ($\mathcal{OML}$) in terms of join ($\vee$), meet ($\wedge$), and complement (c).

$x \vee (y \vee z) = y \vee (x \vee z)$ % AJ
$x \vee (x \wedge y) = x$ % B1
$x \wedge y = c(c(x) \vee c(y))$ % DM
$x \vee (c(x) \wedge (x \vee y)) = x \vee y$ % OM

If we add the orthomodular law OM to the ortholattice 5-basis {AJ, B1, DM, CC, ONE}, then CC and ONE become dependent. The following is an Otter proof.

Proof OML1

1	$c(c(A)) \neq A \mid \$Ans(CC)$	[]
2	$A \vee c(A) \neq B \vee c(B) \mid \$Ans(ONE)$	[]
4	$x \vee (y \vee z) = y \vee (x \vee z)$	[]
5	$x \vee (x \wedge y) = x$	[]
8,7	$x \wedge y = c(c(x) \vee c(y))$	[]
9	$x \vee (c(x) \wedge (x \vee y)) = x \vee y$	[]
10	$x \vee c(c(c(x)) \vee c(x \vee y)) = x \vee y$	[9,8]
12	$x \vee c(c(x) \vee c(y)) = x$	[5:8]

17,16	$x \vee c(c(x)) = x$	[12 → 12]
21,20	$x \vee (y \vee c(c(x))) = y \vee x$	[16 → 4]
27,26	$x \vee c(c(x) \vee y) = x$	[10 → 12]
32	$x \vee (y \vee c(c(x) \vee z)) = y \vee x$	[26 → 4]
34	$x \vee c(y \vee c(x)) = x$	[20 → 26]
36	$x \vee c(c(c(x)) \vee c(y \vee x)) = y \vee x$	[20 → 10,21]
44	$c(x) \vee c(x) = c(x)$	[16 → 34]
67,66	$c(c(x)) \vee x = x$	[44 → 20,17]
76	$c(c(c(c(x)))) \vee x = x$	[66 → 20,17]
84	$c(c(x)) \vee (y \vee x) = y \vee x$	[66 → 4]
91,90	$c(c(c(x))) = c(x)$	[76 → 26,67]
92	$c(c(x)) \vee c(c(c(x)) \vee c(x)) = x$	[76 → 10,91,91,91,91,67]
115,114	$c(c(x)) \vee c(y \vee c(x)) = c(c(x))$	[90 → 34]
119,118	$c(c(x)) = x$	[92:115]
120	$\$Ans(CC)$	[118,1]
127	$x \vee (y \vee x) = y \vee x$	[84:119]
143	$x \vee c(x \vee c(y \vee x)) = y \vee x$	[36:119]
153	$c(x) \vee c(x \vee y) = c(x)$	[118 → 26]
170	$c(c(x) \vee y) \vee x = x$	[26 → 127,27]
203	$c(x \vee y) \vee c(x) = c(x)$	[118 → 170]
232	$x \vee y = y \vee x$	[34 → 32]
239	$(x \vee y) \vee y = x \vee y$	[127 → 232]
245	$x \vee (y \vee z) = (x \vee z) \vee y$	[4 → 232]
268,267	$(x \vee y) \vee x = y \vee x$	[232 → 239]
354,353	$c(x \vee y) \vee c(y \vee x) = c(x \vee y)$	[267 → 153]
411	$c(x \vee y) = c(y \vee x)$	[267 → 203,354]
642,641	$(x \vee y) \vee z = x \vee (y \vee z)$	[232 → 245]
646	$x \vee (y \vee z) = x \vee (z \vee y)$	[267 → 245,642,642,268,268,642]
1231	$x \vee (y \vee c(x)) = c(x) \vee x$	[143 → 32]
1243	$c(x) \vee x = x \vee (y \vee c(x))$	[1231]
1255,1254	$c(x \vee y) \vee (x \vee y) = x \vee (y \vee c(x))$	[153 → 1231,642]
1264,1263	$x \vee (y \vee (z \vee c(x \vee y))) = x \vee (y \vee c(x))$	[641 → 1231,1255]
1472,1471	$c(x \vee y) \vee (y \vee x) = y \vee (x \vee c(y))$	[411 → 1243,642,1264]
1474	$x \vee (y \vee c(x)) = y \vee (x \vee c(y))$	[646 → 1243,1472,642,1264]
1482,1481	$x \vee (y \vee c(x)) = x \vee c(x)$	[232 → 1243]
1485	$x \vee c(x) = y \vee c(y)$	[1474,1482,1482]
1486	$\$Ans(ONE)$	[1485,2]

The following Mace2 jobs show that the $\mathcal{OML}$ 4-basis {AJ, B1, DM, OM} is independent.

Countermodel OMLa

```
x v (x ^ y) = x.                % B1
x ^ y = c(c(x) v c(y)).         % DM
x v (c(x) ^ (x v y)) = x v y.   % OM
A v (B v C) != B v (A v C).     % denial of AJ
```

C: 0 B: 0 A: 1

c:	0	1
	0	0

∧:	0	1
0	0	0
1	0	0

∨:	0	1
0	0	0
1	1	1

Table 8: OMLa.out

Countermodel OMLb

```
x v (y v z) = y v (x v z).        % AJ
x ^ y = c(c(x) v c(y)).           % DM
x v (c(x) ^ (x v y)) = x v y.     % OM
A v (A ^ B) != A.                 % denial of B1
```

B: 0 A: 1

c:	0	1
	0	0

∨:	0	1
0	0	0
1	0	0

∧:	0	1
0	0	0
1	0	0

Table 9: OMLb.out

Countermodel OMLc

```
x v (y v z) = y v (x v z).        % AJ
x v (x ^ y) = x.                  % B1
x v (c(x) ^ (x v y)) = x v y.     % OM
A ^ B != c(c(A) v c(B)).          % denial of DM
```

B: 0 A: 1

c:	0	1
	0	0

∨:	0	1
0	0	0
1	0	1

∧:	0	1
0	0	1
1	1	1

Table 10: OMLc.out

Countermodel OMLd

```
x v (y v z) = y v (x v z).        % AJ
x v (x ^ y) = x.                  % B1
x ^ y = c(c(x) v c(y)).           % DM
A v (c(A) ^ (A v B)) != A v B.    % denial of OM
```

B: 0 A: 1

c:	0	1
	1	1

∨:	0	1
0	0	0
1	0	1

∧:	0	1
0	1	1
1	1	1

Table 11: OMLd.out

2.1.4 Modular Ortholattices

The following is a 6-basis for modular ortholattices ($\mathcal{MOL}$) in terms of join ($\vee$), meet ($\wedge$), and complement (c).

$x \vee (y \vee z) = y \vee (x \vee z)$	% AJ
$x \vee (x \wedge y) = x$	% B1
$x \wedge y = c(c(x) \vee c(y))$	% DM
$c(c(x)) = x$	% CC
$x \vee c(x) = y \vee c(y)$	% ONE
$x \vee (y \wedge (x \vee z)) = x \vee (z \wedge (x \vee y))$	% MOD

This basis is simply our ortholattice 5-basis plus the modularity law MOD. The following Otter proof shows that the orthomodular law OM holds in $\mathcal{MOL}$.

Proof MOL1

3	$x \vee (x \wedge y) = x$	[]
6,5	$x \wedge y = c(c(x) \vee c(y))$	[]
8,7	$c(c(x)) = x$	[]
9	$x \vee c(x) = y \vee c(y)$	[]
10	$x \vee (y \wedge (x \vee z)) = x \vee (z \wedge (x \vee y))$	[]
11	$x \vee c(c(y) \vee c(x \vee z)) = x \vee c(c(z) \vee c(x \vee y))$	[10,6,6]
12	$A \vee (c(A) \wedge (A \vee B)) \neq A \vee B \ \mid \ \$Ans(OM)$	[]
13	$A \vee c(A \vee c(A \vee B)) \neq A \vee B \ \mid \ \$Ans(OM)$	[12,6,8]
14	$x \vee c(c(x) \vee c(y)) = x$	[3:6]
18	$c(x) \vee x = y \vee c(y)$	[7 → 9]
19	$x \vee c(x) = c(y) \vee y$	[18]
40,39	$x \vee c(c(y) \vee y) = x$	[19 → 14]
50	$x \vee c(x \vee c(x \vee y)) = x \vee y$	[19 → 11,40,8,8]
52	$\$Ans(OM)$	[50,13]

A more common modularity law is

$$x \vee (y \wedge (x \vee z)) = (x \vee y) \wedge (x \vee z) \quad \text{\% MOD2}$$

The following are Otter proofs that MOD and MOD2 are equivalent in lattice theory.

Proof ML1

2	$x \vee (y \vee z) = y \vee (x \vee z)$	[]
3	$x \wedge (y \wedge z) = y \wedge (x \wedge z)$	[]
4	$x \vee (x \wedge y) = x$	[]
7,6	$x \wedge (x \vee y) = x$	[]
8	$x \vee (y \wedge (x \vee z)) = x \vee (z \wedge (x \vee y))$	[]
9	$A \vee (B \wedge (A \vee C)) \neq (A \vee B) \wedge (A \vee C) \ \mid \ \$Ans(MOD2)$	[]
10	$(A \vee B) \wedge (A \vee C) \neq A \vee (B \wedge (A \vee C)) \ \mid \ \$Ans(MOD2)$	[9]
13	$x \vee (y \vee (x \wedge z)) = y \vee x$	[4 → 2]
15	$x \wedge x = x$	[4 → 6]
17	$x \wedge (y \vee (x \vee z)) = x$	[2 → 6]
20,19	$x \vee x = x$	[6 → 4]
21	$x \wedge (y \wedge x) = y \wedge x$	[15 → 3]
23	$x \wedge (y \wedge (x \vee z)) = y \wedge x$	[6 → 3]
31	$x \vee ((x \vee y) \vee y) = x \vee y$	[2 → 19]
36,35	$x \wedge (y \vee x) = x$	[19 → 17]
47	$x \vee (y \wedge x) = x$	[19 → 8,7,20]
59	$(x \wedge (y \vee z)) \wedge (y \vee (z \wedge (y \vee x))) = x \wedge (y \vee z)$	[8 → 35]
64,63	$(x \vee y) \vee y = x \vee y$	[35 → 47]
71	$x \vee (x \vee y) = x \vee y$	[31:64]
81	$(x \vee y) \wedge y = y$	[35 → 21,36]
302	$x \vee (y \vee (z \wedge (y \vee x))) = y \vee x$	[8 → 13]
311,310	$(x \vee y) \wedge (x \vee (y \wedge z)) = x \vee (y \wedge z)$	[13 → 81]
356	$x \wedge y = y \wedge x$	[35 → 23]
367	$x \wedge (y \wedge z) = (x \wedge z) \wedge y$	[3 → 356]
369	$(A \vee C) \wedge (A \vee B) \neq A \vee (B \wedge (A \vee C)) \ \mid \ \$Ans(MOD2)$	[356 → 10]
2164	$((x \vee y) \wedge (x \vee z)) \wedge (x \vee (z \wedge (x \vee y))) = (x \vee y) \wedge (x \vee z)$	[71 → 59]
2222,2221	$(x \wedge y) \wedge z = x \wedge (y \wedge z)$	[356 → 367]

2260,2259	$(x \vee y) \wedge (x \vee (z \wedge (x \vee y))) = (x \vee y) \wedge (x \vee z)$	[2164:2222,311]
4218	$(x \vee y) \wedge (x \vee z) = x \vee (z \wedge (x \vee y))$	[302 $\rightarrow$ 81,2260]
4219	$\$Ans(MOD2)$	[4218,369]

Proof ML2

2	$x \vee (y \vee z) = y \vee (x \vee z)$	[]
3	$x \wedge (y \wedge z) = y \wedge (x \wedge z)$	[]
4	$x \vee (x \wedge y) = x$	[]
6	$x \wedge (x \vee y) = x$	[]
8	$x \vee (y \wedge (x \vee z)) = (x \vee y) \wedge (x \vee z)$	[]
10,9	$(x \vee y) \wedge (x \vee z) = x \vee (y \wedge (x \vee z))$	[8]
11	$A \vee (B \wedge (A \vee C)) \neq A \vee (C \wedge (A \vee B)) \mid \$Ans(MOD)$	[]
12	$A \vee (C \wedge (A \vee B)) \neq A \vee (B \wedge (A \vee C)) \mid \$Ans(MOD)$	[11]
19	$x \wedge (y \vee (x \vee z)) = x$	[2 $\rightarrow$ 6]
21	$x \vee x = x$	[6 $\rightarrow$ 4]
25	$x \wedge (y \wedge (x \vee z)) = y \wedge x$	[6 $\rightarrow$ 3]
37	$x \wedge (y \vee x) = x$	[21 $\rightarrow$ 19]
363	$x \wedge y = y \wedge x$	[37 $\rightarrow$ 25]
375	$x \vee (y \wedge (x \vee z)) = x \vee (z \wedge (x \vee y))$	[9 $\rightarrow$ 363,10]
376	$\$Ans(MOD)$	[375,12]

The following Mace2 jobs show that the $\mathcal{MOL}$ 6-basis {AJ, B1, DM, CC, ONE, MOD} is independent.

Countermodel MOLa

```
x v (x ^ y) = x.                          % B1
x ^ y = c(c(x) v c(y)).                   % DM
c(c(x)) = x.                              % CC
x v c(x) = y v c(y).                      % ONE
x v (y ^ (x v z)) = x v (z ^ (x v y)).    % MOD
A v (B v C) != B v (A v C).               % denial of AJ
```

C: 0 B: 0 A: 1

c:	0	1	2
	0	2	1

$\wedge$:	0	1	2
0	0	0	0
1	1	1	0
2	2	0	2

$\vee$:	0	1	2
0	0	0	0
1	1	1	0
2	2	0	2

Table 12: MOLa.out

Countermodel MOLb

```
x v (y v z) = y v (x v z).                % AJ
x ^ y = c(c(x) v c(y)).                   % DM
c(c(x)) = x.                              % CC
x v c(x) = y v c(y).                      % ONE
x v (y ^ (x v z)) = x v (z ^ (x v y)).    % MOD
A v (A ^ B) != A.                         % denial of B1
```

B: 0 A: 1

c:	0	1
	0	1

∨:	0	1
0	0	0
1	0	0

∧:	0	1
0	0	0
1	0	0

Table 13: MOLb.out

Countermodel MOLc

```
x v (y v z) = y v (x v z).                % AJ
x v (x ^ y) = x.                          % B1
c(c(x)) = x.                              % CC
x v c(x) = y v c(y).                      % ONE
x v (y ^ (x v z)) = x v (z ^ (x v y)).    % MOD
A ^ B != c(c(A) v c(B)).                  % denial of DM
```

B: 0 A: 0

c:	0	1
	1	0

∨:	0	1
0	0	0
1	0	1

∧:	0	1
0	1	1
1	1	1

Table 14: MOLc.out

Countermodel MOLd

```
x v (y v z) = y v (x v z).                % AJ
x v (x ^ y) = x.                          % B1
x ^ y = c(c(x) v c(y)).                   % DM
x v c(x) = y v c(y).                      % ONE
x v (y ^ (x v z)) = x v (z ^ (x v y)).    % MOD
c(c(A)) != A.                             % denial of CC
```

A: 0

c:	0	1	2
	1	2	1

∨:	0	1	2
0	0	1	0
1	1	1	1
2	0	1	2

∧:	0	1	2
0	2	2	2
1	2	1	2
2	2	2	2

Table 15: MOLd.out

Countermodel MOLe

```
x v (y v z) = y v (x v z).                % AJ
x v (x ^ y) = x.                          % B1
x ^ y = c(c(x) v c(y)).                   % DM
c(c(x)) = x.                              % CC
x v (y ^ (x v z)) = x v (z ^ (x v y)).    % MOD
A v c(A) != B v c(B).                     % denial of ONE
```

B: 0 A: 1

c:	0	1	2
	2	1	0

∨:	0	1	2
0	0	0	0
1	0	1	1
2	0	1	2

∧:	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

Table 16: MOLe.out

Countermodel MOLf

```
x v (y v z) = y v (x v z).                % AJ
x v (x ^ y) = x.                          % B1
x ^ y = c(c(x) v c(y)).                   % DM
c(c(x)) = x.                              % CC
x v c(x) = y v c(y).                      % ONE
x v c(x) = 1.                             % ONEa
c(1) = 0.                                 % DEF_0
A v (B ^ (A v C)) != A v (C ^ (A v B)).   % denial of MOD
```

C: 2 B: 3 A: 4

c:	0	1	2	3	4	5
	1	0	3	2	4	5

$\vee$:	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	1	1	1	1	1
2	2	1	2	1	1	5
3	3	1	1	3	3	1
4	4	1	1	3	4	1
5	5	1	5	1	1	5

$\wedge$:	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	2	0	0	2
3	0	3	0	3	4	0
4	0	4	0	4	4	0
5	0	5	2	0	0	5

Table 17: MOLf.out

2.1.5 Boolean Algebra

The following is a 4-basis for Boolean algebra ($\mathcal{BA}$) in terms of join ($\vee$), meet ($\wedge$), and complement (c).

$x \vee (y \vee z) = y \vee (x \vee z)$	% AJ
$x \wedge y = c(c(x) \vee c(y))$	% DM
$x \vee c(x) = y \vee c(y)$	% ONE
$(x \vee c(y)) \wedge (x \vee y) = x$	% CUT

The following are proofs of distributivity, modularity, CC, and B1 from the $\mathcal{BA}$ 4-basis.

Proof BA1

2	$x \vee (y \vee z) = y \vee (x \vee z)$	[]
3	$x \wedge y = c(c(x) \vee c(y))$	[]
4	$c(c(x) \vee c(y)) = x \wedge y$	[3]
7	$(x \vee c(y)) \wedge (x \vee y) = x$	[]
10,9	$x \vee c(x) = 1$	[]
12,11	$c(1) = 0$	[]
13	$A \wedge (B \vee C) \neq (A \wedge B) \vee (A \wedge C) \mid \$Ans(DIST1)$	[]
14	$x \vee (y \vee (z \vee u)) = z \vee (x \vee (y \vee u))$	[2 $\rightarrow$ 2]
15	$x \vee (y \vee (z \vee u)) = y \vee (z \vee (x \vee u))$	[14]
17,16	$1 \vee 0 = 1$	[11 $\rightarrow$ 9]
18	$x \vee (y \vee c(x)) = y \vee 1$	[9 $\rightarrow$ 2]
20	$x \vee 1 = 1 \vee (x \vee 0)$	[16 $\rightarrow$ 2]
21	$1 \vee (x \vee 0) = x \vee 1$	[20]
22	$c(0 \vee c(x)) = 1 \wedge x$	[11 $\rightarrow$ 4]
24	$c((x \wedge y) \vee c(z)) = (c(x) \vee c(y)) \wedge z$	[4 $\rightarrow$ 4]
27,26	$c(c(x) \vee 0) = x \wedge 1$	[11 $\rightarrow$ 4]

29,28	$c(c(x) \vee (y \wedge z)) = x \wedge (c(y) \vee c(z))$	[4 → 4]
30	$x \wedge c(x) = 0$	[9 → 4,12]
32	$(c(x) \vee c(y)) \vee (x \wedge y) = 1$	[4 → 9]
34	$1 \wedge 0 = 0$	[11 → 30]
36	$(c(x) \vee c(y)) \wedge (x \wedge y) = 0$	[4 → 30]
38	$x \vee (1 \vee (y \vee 0)) = y \vee (x \vee 1)$	[20 → 2]
39	$x \vee (y \vee 1) = y \vee (1 \vee (x \vee 0))$	[38]
41,40	$(x \vee 0) \wedge (x \vee 1) = x$	[11 → 7]
42	$(x \vee (y \wedge z)) \wedge (x \vee (c(y) \vee c(z))) = x$	[4 → 7]
45,44	$1 \wedge (x \vee x) = x$	[9 → 7]
46	$(x \vee 0) \wedge (1 \vee (x \vee 0)) = x$	[20 → 7,12]
48	$(1 \vee c(0)) \wedge 1 = 1$	[16 → 7]
50	$(x \vee c(c(x))) \wedge 1 = x$	[9 → 7]
52	$(x \vee c(y \vee z)) \wedge (y \vee (x \vee z)) = x$	[2 → 7]
55,54	$1 \wedge (x \vee ((x \vee y) \vee y)) = x \vee y$	[2 → 44]
58	$x \vee (y \vee 1) = 1 \vee (x \vee (y \vee 0))$	[21 → 2]
59	$1 \vee (x \vee (y \vee 0)) = x \vee (y \vee 1)$	[58]
63,62	$c(0 \vee 0) = 1 \wedge 1$	[11 → 22]
64	$c(0 \vee (x \wedge y)) = 1 \wedge (c(x) \vee c(y))$	[4 → 22]
66	$(x \vee (1 \wedge y)) \wedge (x \vee (0 \vee c(y))) = x$	[22 → 7]
70	$(0 \vee c(x)) \vee (1 \wedge x) = 1$	[22 → 9]
72	$(x \vee (1 \wedge 1)) \wedge (x \vee (0 \vee 0)) = x$	[62 → 7]
85,84	$c((x \wedge y) \vee 0) = (c(x) \vee c(y)) \wedge 1$	[4 → 26]
90	$(c(x) \vee 0) \vee (x \wedge 1) = 1$	[26 → 9]
94	$x \vee (y \vee (z \vee c(x))) = y \vee (z \vee 1)$	[9 → 14]
98	$x \vee (y \vee (z \vee u)) = z \vee (y \vee (x \vee u))$	[2 → 14]
105	$(x \vee c(y \vee (z \vee u))) \wedge (z \vee (x \vee (y \vee u))) = x$	[14 → 7]
127	$1 \wedge (x \vee (y \vee ((x \vee (y \vee z)) \vee z))) = x \vee (y \vee z)$	[15 → 44]
130	$(x \vee c(y \vee (z \vee u))) \wedge (y \vee (z \vee (x \vee u))) = x$	[15 → 7]
137	$(c(x) \vee 0) \vee (y \vee (x \wedge 1)) = y \vee 1$	[26 → 18]
141	$(c(x) \vee c(y)) \vee (z \vee (x \wedge y)) = z \vee 1$	[4 → 18]
161,160	$c(x \vee c(y)) = (c(x \vee 0) \vee c(x \vee 1)) \wedge y$	[40 → 24]
163,162	$((1 \wedge 1) \vee c(0 \vee 1)) \wedge x = 1 \wedge x$	[34 → 24,161,63,12,10]
174	$(0 \vee (1 \wedge 1)) \vee 0 = 1$	[62 → 70,45]
178	$(0 \vee c(x \vee x)) \vee x = 1$	[44 → 70]
182	$1 \wedge ((0 \vee (1 \wedge 1)) \vee 1) = 0 \vee (1 \wedge 1)$	[174 → 40]
185,184	$(0 \vee (1 \wedge 1)) \vee 1 = 1 \vee 1$	[174 → 21]
191	$0 \vee (1 \wedge 1) = 1$	[182:185,45]
196,195	$c((x \wedge 1) \vee (y \wedge z)) = (c(x) \vee 0) \wedge (c(y) \vee c(z))$	[26 → 28]
197	$c(c(x) \vee 1) = x \wedge (((0 \vee c(1 \vee 1)) \wedge 0) \vee 0)$	[48 → 28,161,17,12,12]
199	$c(c(x) \vee y) = x \wedge (0 \vee c(y \vee y))$	[44 → 28,12]
200	$c(c(x) \vee y) = x \wedge (c(y \vee 0) \vee c(y \vee 1))$	[40 → 28]
201	$x \wedge (0 \vee c(y \vee y)) = c(c(x) \vee y)$	[199]
202	$x \wedge (c(y \vee 0) \vee c(y \vee 1)) = c(c(x) \vee y)$	[200]
205	$(c(x) \vee (y \wedge z)) \vee (x \wedge (c(y) \vee c(z))) = 1$	[28 → 9]
210	$(0 \vee c(1 \wedge 1)) \wedge 1 = 0$	[191 → 7]
212	$0 \vee (x \vee (1 \wedge 1)) = x \vee 1$	[191 → 2]
226	$(((c(x \vee 0) \vee c(x \vee 1)) \wedge c(x)) \vee 0) \vee x = 1$	[50 → 90,161]
234	$((x \wedge 1) \vee c(y)) \vee ((c(x) \vee 0) \wedge y) = 1$	[26 → 32]
240	$(c(x \vee 0) \vee c(x \vee 1)) \vee x = 1$	[40 → 32]
250	$x \vee ((0 \vee c((x \vee y) \vee (x \vee y))) \vee y) = 1$	[2 → 178]
255	$x \vee (y \vee 1) = (0 \vee c(z \vee z)) \vee (x \vee (y \vee z))$	[178 → 14]
258	$x \vee 1 = (0 \vee c(y \vee y)) \vee (x \vee y)$	[178 → 2]
260	$(0 \vee c(x \vee x)) \vee (y \vee (z \vee x)) = y \vee (z \vee 1)$	[255]

263,262	$((1 \wedge (1 \wedge 1)) \vee 0) \vee 0 = 1$	[210 → 90,161,63,163]
278	$c(c(x) \vee y) = x \wedge (c(y \vee 0) \vee c(1 \vee (y \vee 0)))$	[46 → 28]
287,286	$(0 \vee 0) \vee 1 = 0 \vee 1$	[90 → 212,12]
306	$(x \vee c(1 \vee (y \vee 0))) \wedge (y \vee (x \vee 1)) = x$	[38 → 7]
312,311	$(1 \wedge (1 \wedge 1)) \vee 0 = 1$	[262 → 46,263,45]
314	$1 \wedge (1 \wedge 1) = 1$	[311 → 46,312,45]
322	$x \vee (y \vee 1) = x \vee (1 \vee (y \vee 0))$	[2 → 39]
325	$x \vee (1 \vee (y \vee 0)) = x \vee (y \vee 1)$	[322]
334	$(x \vee c(y \vee 1)) \wedge (y \vee (1 \vee (x \vee 0))) = x$	[39 → 7]
376	$(x \vee y) \wedge (x \vee (0 \vee c(y \vee y))) = x$	[44 → 42,12]
380	$1 \wedge (0 \vee (0 \vee 0)) = 0$	[191 → 42,12,12]
388	$(x \vee (y \wedge (c(z) \vee 0))) \wedge (x \vee (c(y) \vee (z \wedge 1))) = x$	[26 → 42]
390	$(x \vee (y \wedge x)) \wedge (c(y) \vee 1) = x$	[18 → 42]
392	$(x \vee (y \wedge z)) \wedge (c(y) \vee (x \vee c(z))) = x$	[2 → 42]
399,398	$(0 \vee c(0 \vee (0 \vee 0))) \vee 0 = 1$	[380 → 70]
422	$(x \vee (1 \wedge 1)) \wedge (0 \vee (x \vee 0)) = x$	[62 → 52]
424	$(x \vee (y \wedge 1)) \wedge (c(y) \vee (x \vee 0)) = x$	[26 → 52]
440	$(x \vee c((0 \vee c((x \vee y) \vee (x \vee y))) \vee y)) \wedge 1 = x$	[178 → 52]
444	$(1 \vee c(x \vee (y \vee 0))) \wedge (y \vee (x \vee 1)) = 1$	[38 → 52]
448	$(x \vee c(y \vee (z \vee u))) \wedge (z \vee (y \vee (x \vee u))) = x$	[14 → 52]
466	$((x \vee x) \vee x) \wedge (0 \vee 1) = x \vee x$	[44 → 390,12]
500	$1 \wedge (x \vee (y \vee ((y \vee (x \vee z)) \vee z))) = y \vee (x \vee z)$	[14 → 54]
505	$0 \vee c(0 \vee (0 \vee 0)) = 1$	[398 → 46,399,45]
520	$0 \vee (x \vee c(0 \vee (0 \vee 0))) = x \vee 1$	[505 → 2]
561,560	$(0 \vee c((x \vee 0) \vee (x \vee 0))) \vee (x \vee 1) = 1 \vee 1$	[178 → 59]
580	$1 \wedge (c(x \vee 0) \vee c(1 \vee (x \vee 0))) = c(0 \vee x)$	[46 → 64]
582	$c(0 \vee x) = 1 \wedge (0 \vee c(x \vee x))$	[44 → 64,12]
584	$1 \wedge (0 \vee c(x \vee x)) = c(0 \vee x)$	[582]
618	$(x \vee (1 \wedge y)) \wedge (0 \vee (x \vee c(y))) = x$	[2 → 66]
631,630	$((1 \wedge 1) \vee c(0 \vee 1)) \vee 0 = 1$	[62 → 240]
643	$(1 \wedge 1) \vee c(0 \vee 1) = 1$	[240 → 46,63,631,45,63]
658	$c(x \vee 0) = (0 \vee c(x \vee x)) \wedge 1$	[44 → 84,12]
662	$(0 \vee c(x \vee x)) \wedge 1 = c(x \vee 0)$	[658]
665	$(x \vee ((c(y) \vee c(z)) \wedge 1)) \wedge ((y \wedge z) \vee (x \vee 0)) = x$	[84 → 52]
669	$((x \wedge y) \vee 0) \wedge ((c(x) \vee c(y)) \wedge 1) = 0$	[84 → 30]
683	$(1 \wedge 1) \vee c(1 \vee (0 \vee 0)) = 1$	[20 → 643]
685	$1 \wedge (0 \vee ((1 \wedge 1) \vee 1)) = 1 \wedge 1$	[643 → 52]
688,687	$(1 \wedge 1) \vee 1 = (0 \vee 1) \vee 1$	[643 → 18]
703,702	$1 \wedge 1 = 0 \vee 1$	[685:688,55]
710	$(0 \vee 1) \vee c(1 \vee (0 \vee 0)) = 1$	[683:703]
732	$(x \vee (0 \vee 1)) \wedge (0 \vee (x \vee 0)) = x$	[422:703]
755,754	$1 \wedge (0 \vee 1) = 1$	[314:703]
772	$0 \vee (x \vee (0 \vee 1)) = x \vee 1$	[212:703]
778	$0 \vee (0 \vee 1) = 1$	[191:703]
788	$(x \vee (0 \vee 1)) \wedge (x \vee (0 \vee 0)) = x$	[72:703]
791,790	$c(0 \vee 0) = 0 \vee 1$	[62:703]
792	$(1 \vee (0 \vee 1)) \wedge (0 \vee 1) = 1$	[702 → 390,12]
794	$c(c(x) \vee (0 \vee 1)) = x \wedge (0 \vee 0)$	[702 → 28,12,12]
806	$0 \vee (1 \vee (0 \vee 0)) = 1$	[20 → 778]
811,810	$0 \vee (0 \vee (x \vee 1)) = x \vee 1$	[778 → 15]
814	$c((0 \vee 1) \vee (x \wedge y)) = (0 \vee 0) \wedge (c(x) \vee c(y))$	[790 → 28]
840	$0 \vee (1 \vee (x \vee (0 \vee 0))) = x \vee 1$	[806 → 15]
860	$0 \vee (0 \vee (1 \vee (x \vee 0))) = x \vee 1$	[39 → 772]
862	$x \vee 1 = 0 \vee (0 \vee (1 \vee (x \vee 0)))$	[860]

866,865	$(0 \vee (1 \vee 1)) \wedge (0 \vee 1) = 1$	[39 → 792,17]
925,924	$(1 \vee (0 \vee 0)) \vee 1 = (0 \vee 1) \vee 1$	[710 → 18]
1073,1072	$1 \wedge (0 \vee c((x \vee y) \vee (x \vee y))) = c(x \vee (0 \vee y))$	[2 → 582]
1187	$(0 \vee (x \vee 1)) \wedge (0 \vee (x \vee 0)) = x$	[2 → 732]
1211	$(x \vee c(y \vee (z \vee c(x)))) \wedge (z \vee (y \vee 1)) = x$	[18 → 105]
1229	$(0 \vee (x \vee 1)) \wedge (x \vee (0 \vee 0)) = x$	[2 → 788]
1232,1231	$0 \vee c((0 \vee 0) \vee (0 \vee 0)) = (1 \vee 1) \wedge 1$	[178 → 788,561]
1294,1293	$(c((0 \vee 0) \vee 0) \vee c(0 \vee 1)) \vee 1 = 0 \vee (1 \vee 1)$	[240 → 840,287]
1296,1295	$((1 \vee 1) \wedge 1) \vee 1 = 0 \vee (1 \vee 1)$	[178 → 840,1232]
1318	$c(x \vee 1) = c(0 \vee (0 \vee (1 \vee (x \vee 0))))$	[860 → 582,1073]
1325,1324	$0 \vee (0 \vee (1 \vee (x \vee 0))) = 1 \vee (x \vee 0)$	[20 → 862]
1327	$c(x \vee 1) = c(1 \vee (x \vee 0))$	[1318:1325]
1440	$c((0 \vee 0) \vee 0) \vee c(0 \vee 1) = (1 \vee 1) \wedge 1$	[240 → 1229,287,1294,811,287]
1589,1588	$c(1 \vee (c(x) \vee 0)) = x \wedge (0 \vee c(1 \vee 1))$	[20 → 199]
1624	$c((1 \wedge c(x \vee x)) \vee x) = 0$	[7 → 201,161,791,10]
1683	$((1 \wedge c(x \vee x)) \vee x) \vee 0 = 1$	[1624 → 9]
1704,1703	$x \vee (((1 \wedge c(y \vee y)) \vee y) \vee 1) = x \vee (1 \vee 1)$	[1683 → 325]
1705	$(1 \wedge c(x \vee x)) \vee x = 1$	[1683 → 1187,1704,866]
1755	$x \vee 1 = (1 \wedge c(y \vee y)) \vee (x \vee y)$	[1705 → 2]
1762	$(1 \wedge c(x \vee x)) \vee (y \vee x) = y \vee 1$	[1755]
1867	$(x \vee y) \wedge (0 \vee (x \vee c(y \vee y))) = x$	[2 → 376]
1877	$(x \vee ((c(y) \vee 0) \wedge 1)) \wedge ((y \wedge 1) \vee (x \vee 0)) = x$	[26 → 424]
1911	$(0 \vee (0 \vee 0)) \vee (x \vee 1) = 0 \vee (x \vee 1)$	[520 → 94]
2367	$(c(x) \vee (1 \wedge y)) \vee (x \wedge (0 \vee c(y))) = 1$	[11 → 205]
2369	$(0 \vee (x \wedge x)) \vee c(x) = 1$	[44 → 205,12]
2378,2377	$(0 \vee (x \wedge x)) \vee 1 = x \vee 1$	[2369 → 18]
2396	$(0 \vee (x \wedge x)) \vee (y \vee c(x)) = y \vee 1$	[2369 → 2]
2403,2402	$1 \vee ((0 \vee (x \wedge x)) \vee 0) = x \vee 1$	[20 → 2377]
2462,2461	$x \vee (((1 \wedge c(0)) \vee 0) \vee 1) = x \vee (1 \vee 1)$	[226 → 325,791,10]
2463	$(1 \wedge c(0)) \vee 0 = 1$	[226 → 1187,791,10,2462,866,791,10]
2468,2467	$(1 \wedge c(0)) \vee 1 = 1 \vee 1$	[2463 → 1762,791,755]
2479	$1 \wedge c(0) = 1$	[2463 → 1187,2468,866]
2484,2483	$(c(0) \vee 1) \wedge (0 \vee 1) = c(0)$	[2479 → 390,12]
2485	$(x \vee 1) \wedge (0 \vee (x \vee c(c(0)))) = x$	[2479 → 618]
2532	$(1 \vee (c(0) \vee 0)) \wedge (0 \vee 1) = c(0)$	[20 → 2483]
2564	$0 \vee (c(0) \wedge c(0)) = c(0)$	[2369 → 2485,2378,2484]
2583	$(0 \vee c(c(0) \wedge c(0))) \wedge c(0) = 0$	[2564 → 7]
2605	$((0 \wedge (0 \vee c(1 \vee 1))) \vee c(0 \vee 1)) \wedge 1 = 0 \wedge 1$	[2532 → 84,27,1589]
2613	$((0 \wedge (0 \vee c(1 \vee 1))) \vee c(0 \vee 1)) \vee c(0) = 1$	[2532 → 32,1589]
3123	$(x \vee ((c(y) \vee 0) \wedge z)) \wedge ((y \wedge 1) \vee (x \vee c(z))) = x$	[26 → 392]
3344	$0 \vee ((x \vee 1) \wedge (x \vee 1)) = x \vee 1$	[2369 → 334,2403,55]
3353,3352	$(c(x \vee 1) \vee c(x \vee 1)) \vee (x \vee 1) = 0 \vee 1$	[3344 → 141]
3541	$(0 \vee (0 \vee 0)) \vee (0 \vee (1 \vee 1)) = 1 \vee 1$	[1295 → 1911,1296,811]
3895	$((1 \wedge c(x)) \vee (1 \wedge x)) \vee 0 = 1$	[7 → 2367,161,791,10]
3924,3923	$((1 \wedge c(x)) \vee (1 \wedge x)) \vee 1 = 1 \vee 1$	[3895 → 1762,791,755]
3928	$(1 \wedge c(x)) \vee (1 \wedge x) = 1$	[3895 → 1187,3924,866]
3964	$(1 \wedge x) \vee c(0 \vee x) = 1$	[584 → 3928,161,791,10,45]
4033	$x \vee c(0 \vee (x \vee x)) = 1$	[44 → 3964]
4112,4111	$(1 \wedge x) \vee 1 = (0 \vee x) \vee 1$	[3964 → 2396,2378]
4204	$x \vee c(x \vee (0 \vee x)) = 1$	[2 → 4033]
4218,4217	$1 \wedge (0 \vee (x \vee (x \vee x))) = x$	[4033 → 130]
4219	$1 \wedge (x \vee (x \vee (0 \vee x))) = x$	[4033 → 105]
4228	$(0 \vee (x \vee x)) \vee 1 = x \vee 1$	[4033 → 2396,2378]
4300,4299	$(x \vee (0 \vee x)) \vee 1 = x \vee 1$	[4204 → 2396,2378]

4437,4436	$1 \wedge (c(0) \vee (c(0) \vee 1)) = c(0)$	[94 → 4217]
4474	$1 \wedge ((1 \vee (0 \vee 0)) \vee ((0 \vee 1) \vee 1)) = 1 \vee (0 \vee 0)$	[806 → 4219,925]
4554	$(c(0) \vee 1) \vee 1 = c(0) \vee 1$	[18 → 4228]
4567,4566	$0 \vee (1 \vee 1) = 1$	[4228 → 127,4218]
4590,4589	$1 \vee 1 = 0 \vee 1$	[3541:4567,4300]
4878	$((0 \wedge (0 \vee c(0 \vee 1))) \vee c(0 \vee 1)) \vee c(0) = 1$	[2613:4590]
4884	$((0 \wedge (0 \vee c(0 \vee 1))) \vee c(0 \vee 1)) \wedge 1 = 0 \wedge 1$	[2605:4590]
4945,4944	$c(1 \vee (c(x) \vee 0)) = x \wedge (0 \vee c(0 \vee 1))$	[1588:4590]
4948	$c((0 \vee 0) \vee 0) \vee c(0 \vee 1) = (0 \vee 1) \wedge 1$	[1440:4590]
4976	$c(c(x) \vee 1) = x \wedge (((0 \vee c(0 \vee 1)) \wedge 0) \vee 0)$	[197:4590]
4982	$(0 \vee c(x \vee x)) \vee (1 \vee x) = 0 \vee 1$	[258 → 4589]
5310,5309	$c(0) \vee 1 = c(0)$	[4554 → 54,4437]
5347	$c(c(0)) = 0 \wedge (0 \vee c(0 \vee 1))$	[4554 → 1327,5310,5310,4945]
5358,5357	$c(0) = 0 \vee 1$	[4554 → 810,5310,10,5310,5310]
5406,5405	$0 \wedge (0 \vee c(0 \vee 1)) = c(0 \vee 1)$	[5347:5358]
5424,5423	$0 \vee 1 = 1$	[4878:5406,5358,3353]
5440,5439	$(0 \vee 0) \wedge 1 = 0$	[2583:5358,5424,5358,5424,703,5424,12,5358,5424]
5448	$((0 \wedge (0 \vee 0)) \vee 0) \wedge 1 = 0 \wedge 1$	[4884:5424,12,5424,12]
5456,5455	$1 \vee (0 \vee 0) = 1$	[4474:5424,4590,5424,925,5424,4590,5424,703,5424]
5466,5465	$0 \wedge (0 \vee 0) = 0$	[5405:5424,12,5424,12]
5476,5475	$c(0) = 1$	[5357:5424]
5499	$(0 \vee c(x \vee x)) \vee (1 \vee x) = 1$	[4982:5424]
5504,5503	$c(c(x) \vee 1) = x \wedge (((0 \vee 0) \wedge 0) \vee 0)$	[4976:5424,12]
5531	$c((0 \vee 0) \vee 0) \vee 0 = 1$	[4948:5424,12,5424,703,5424]
5535	$c(1 \vee (c(x) \vee 0)) = x \wedge (0 \vee 0)$	[4944:5424,12]
5652,5651	$1 \vee 1 = 1$	[4589:5424]
5838,5837	$c(1 \vee (x \wedge y)) = (0 \vee 0) \wedge (c(x) \vee c(y))$	[814:5424]
5840,5839	$x \wedge (((0 \vee 0) \wedge 0) \vee 0) = x \wedge (0 \vee 0)$	[794:5424,5504]
5842,5841	$c(0 \vee 0) = 1$	[790:5424]
5862,5861	$1 \wedge 1 = 1$	[702:5424]
5874,5873	$((x \vee x) \vee x) \wedge 1 = x \vee x$	[466:5424]
5913,5912	$0 \wedge 1 = 0$	[5448:5466,5440]
5948	$c(c(x) \vee 1) = x \wedge (0 \vee 0)$	[5503:5840]
5966	$(x \vee (0 \wedge x)) \wedge 1 = x$	[5475 → 390,5652]
6031,6030	$(0 \vee 0) \wedge (c(x) \vee c(y)) = 0 \wedge (c(x) \vee c(y))$	[5475 → 28,5838]
6053,6052	$c(1 \vee (x \wedge y)) = 0 \wedge (c(x) \vee c(y))$	[5837:6031]
6257,6256	$1 \wedge (c(x \vee (0 \wedge x)) \vee 0) = c(0 \vee x)$	[5966 → 64,12]
6383,6382	$0 \vee 0 = 0$	[5531 → 278,12,5842,5456,12,17,5874]
6391,6390	$c(c(x) \vee 1) = x \wedge 0$	[5948:6383]
6437,6436	$c(1 \vee (c(x) \vee 0)) = x \wedge 0$	[5535:6383]
6499,6498	$0 \vee (x \vee 0) = x \vee 0$	[6382 → 2]
6638	$(1 \wedge (x \vee 0)) \vee c(x \vee 0) = 1$	[6498 → 3964]
6782	$c(1 \vee ((x \wedge 0) \vee 0)) = (1 \vee (c(x) \vee 0)) \wedge 0$	[6436 → 6436]
6784	$(1 \vee (x \wedge 0)) \wedge (c(x) \vee 1) = 1$	[6436 → 444,5652]
7014	$(0 \vee (((x \wedge 1) \vee (x \wedge 0)) \wedge x)) \vee 1 = x \vee 1$	[5499 → 94,161,27,6391,5652]
7283	$1 \wedge ((x \wedge 1) \vee (x \wedge 0)) = 1 \wedge x$	[278 → 580,6383,5476,6383,17,12,17,6437,161,6383,5476,5424,12,17]
7345	$(1 \wedge (c(x) \vee 0)) \vee (x \wedge 1) = 1$	[278 → 6638,6383,5476,6383,17,12,17]
7391	$(x \vee 1) \wedge (x \vee ((0 \wedge (c(y) \vee 1)) \vee (y \wedge 0))) = x$	[6784 → 42,6053,5476,6391]
7679	$c(0 \vee x) \vee x = 1$	[5966 → 7345,6257]
7693	$c(x \vee 0) \vee (x \vee 0) = 1$	[6498 → 7679]
7743	$(1 \wedge x) \vee c(x) = 1$	[160 → 7679,6383,5476,5424,12,17]
7808,7807	$(x \vee x) \wedge 1 = x$	[7679 → 1867,161,6383,5476,5424,12,17,45,5424,161,6383,5476,5424,12,17,45]

7824	$(0 \vee c(x \vee x)) \vee 1 = c(0 \vee (y \vee x)) \vee (y \vee 1)$	[7679 → 260]
7920	$c(x \vee 0) \vee (x \vee 1) = 1$	[7679 → 59,5652,6499]
7949,7948	$(0 \vee x) \vee 1 = x \vee 1$	[7679 → 2396,2378,161,6383,5476,5424,12,17,4112]
8055	$c(x \vee x) \vee 1 = c(0 \vee (y \vee x)) \vee (y \vee 1)$	[7824:7949]
8075,8074	$(((x \wedge 1) \vee (x \wedge 0)) \wedge x) \vee 1 = x \vee 1$	[7014:7949]
8136	$(x \vee x) \vee 1 = x \vee 1$	[4228:7949]
8141,8140	$(1 \wedge x) \vee 1 = x \vee 1$	[4111:7949]
8152	$(x \wedge x) \vee 1 = x \vee 1$	[2377:7949]
8207	$(1 \wedge (c(x \vee x) \vee 0)) \vee x = 1$	[7807 → 7345]
8250	$c(x \vee 0) = (c(x \vee x) \vee 0) \wedge 1$	[7807 → 84,12]
8260	$(c(x \vee x) \vee 0) \wedge 1 = c(x \vee 0)$	[8250]
8288	$x \vee c(x \vee x) = 1$	[44 → 7743]
8349	$x \wedge (c(x) \vee c(x \vee 1)) = 0$	[8288 → 200,12,161,27,6391,85,196,5476,41,7808,161,27,6391,8075]
8353,8352	$1 \wedge (x \vee ((x \vee y) \vee ((x \vee y) \vee y))) = x \vee y$	[8288 → 448]
8359	$1 \wedge (x \vee (x \vee x)) = x$	[8288 → 52]
8565,8564	$(x \vee 1) \vee (x \vee 1) = x \vee 1$	[8136 → 500,8353]
8727,8726	$x \wedge (c(y) \vee c(y \vee 1)) = x \wedge (c(y) \vee c(y))$	[8152 → 202,85,7808,29]
8779	$x \wedge (c(x) \vee c(x)) = 0$	[8349:8727]
8821	$x \vee c(x \vee (x \vee x)) = 1$	[8359 → 7743]
8851	$(c(x) \vee 1) \wedge ((x \wedge 0) \vee (x \wedge 0)) = 0$	[6390 → 8779,6391]
8877	$(x \wedge 1) \vee (c(x) \vee 0) = 1$	[278 → 7693,6383,5476,6383,17,12,17]
8893	$(x \vee c(c(x \vee 0) \vee (y \vee 0))) \wedge (y \vee 1) = x$	[7693 → 448]
9085	$(x \wedge 1) \vee (c(x) \vee 1) = 1$	[278 → 7920,6383,5476,6383,17,12,17]
9089	$c(x) \vee (x \vee 1) = 1$	[8152 → 7920,85,7808]
9110,9109	$c(x \vee 1) \vee 1 = c(x \vee 0) \vee 1$	[7920 → 1762,8565,8141]
9199	$c(0 \vee x) \vee (x \vee 1) = 1$	[7948 → 9089]
9221	$x \vee (c(x) \vee 1) = 1$	[2 → 9089]
9225	$(x \vee (x \wedge 0)) \wedge 1 = x$	[9089 → 306,6437]
9228,9227	$c(x \vee 0) \vee 1 = c(x) \vee 1$	[9089 → 1762,8565,8141,9110]
9380,9379	$(x \wedge 0) \vee 1 = x \vee 1$	[9221 → 1762,8565,6391,8141]
9397	$(x \vee c(y \vee (z \vee (c(x) \vee 1)))) \wedge (y \vee (z \vee 1)) = x$	[9221 → 130]
9495	$1 \wedge (c(x \vee (x \wedge 0)) \vee 0) = c(0 \vee x)$	[9225 → 64,12]
9516,9515	$1 \vee ((x \wedge 0) \vee 0) = x \vee 1$	[20 → 9379]
9518,9517	$c(x \vee 1) = (1 \vee (c(x) \vee 0)) \wedge 0$	[6782:9516]
9616,9615	$(1 \vee (c(c(x)) \vee 0)) \wedge 0 = x \wedge 0$	[6390:9518]
10144	$(x \vee (y \wedge (x \vee (x \vee x)))) \wedge (c(y) \vee 1) = x$	[8821 → 392]
10354	$x \vee 1 = (y \wedge 1) \vee (c(y) \vee (x \vee 0))$	[8877 → 15]
10383	$(x \wedge 1) \vee (c(x) \vee (y \vee 0)) = y \vee 1$	[10354]
10605	$(x \wedge 1) \vee 1 = x \vee 1$	[9085 → 1762,8565,9518,9616,8141,9380]
10676,10675	$c(x \vee x) \vee 1 = c(x) \vee 1$	[662 → 10605,9228,7949]
10714,10713	$c(0 \vee (x \vee y)) \vee (x \vee 1) = c(y) \vee 1$	[8055:10676]
12015,12014	$c(x) \vee 1 = 1$	[8136 → 9199,10714]
12035,12034	$x \vee 1 = 1$	[2 → 9199,12015]
12056	$(x \vee (y \wedge (x \vee (x \vee x)))) \wedge 1 = x$	[10144:12035]
12059,12058	$(x \vee 0) \wedge 1 = x$	[9397:12035,12035,12035,12,12035,12035]
12067,12066	$x \wedge 0 = 0$	[8851:12035,45]
12073,12072	$1 \wedge (x \vee 0) = x$	[7391:12035,12035,5913,12067,6383]
12098	$(x \vee (y \wedge x)) \wedge 1 = x$	[390:12035]
12162	$(x \wedge 1) \vee (c(x) \vee (y \vee 0)) = 1$	[10383:12035]
12288	$(x \vee c(c(x \vee 0) \vee (y \vee 0))) \wedge 1 = x$	[8893:12035]
12490	$(x \vee c(y \vee (z \vee c(x)))) \wedge 1 = x$	[1211:12035,12035]
12518	$(c(x) \vee 0) \vee (y \vee (x \wedge 1)) = 1$	[137:12035]
12524	$c(x \vee x) = c(x \vee 0)$	[8260:12059]
12566	$(x \vee c(y)) \wedge ((y \wedge 1) \vee (x \vee 0)) = x$	[1877:12059]

12607	$c(x \vee 0) = c(0 \vee x)$	[9495:12067,12073]
12629	$x \wedge 1 = 1 \wedge x$	[7283:12067,12073]
12680	$c(x \vee x) \vee x = 1$	[8207:12073]
12761	$1 \wedge x = x \wedge 1$	[12629]
12927,12926	$c(x \vee (y \wedge z)) = (c(x \vee 0) \vee 0) \wedge (c(y) \vee c(z))$	[12058 → 195]
12932	$c((x \wedge 1) \vee y) = (c(x) \vee 0) \wedge (c(y \vee 0) \vee 0)$	[12058 → 195,12]
12958	$(c(x) \vee 0) \wedge (c(y \vee 0) \vee 0) = c((x \wedge 1) \vee y)$	[12932]
12967	$x \vee c(x \vee 0) = 1$	[12072 → 7743]
13050	$1 \wedge (x \vee c((0 \vee c((x \vee y) \vee (x \vee y))) \vee y)) = x$	[440 → 12629]
13191	$(x \vee (x \vee x)) \wedge 1 = x$	[8359 → 12761]
13197	$(0 \vee (x \vee (x \vee x))) \wedge 1 = x$	[4217 → 12761]
13253	$(x \vee (y \wedge 1)) \wedge (x \vee (0 \vee c(y))) = x$	[12761 → 66]
13479	$(x \vee (((x \vee c(y)) \vee (x \vee c(y))) \wedge y)) \wedge 1 = x$	[12680 → 392]
13483	$(x \vee (((y \wedge 1) \vee (y \wedge 1)) \wedge (c(y) \vee 0))) \wedge 1 = x$	[12680 → 388,12035]
13491	$(x \vee c(c((x \vee y) \vee (x \vee y)) \vee (z \vee y))) \wedge 1 = x$	[12680 → 448,12035]
13644,13643	$x \wedge (c(x) \vee 0) = 0$	[12967 → 669,5862,12059]
13707	$(x \vee (x \wedge 1)) \wedge 1 = x$	[12761 → 12098]
13751	$(c(x \vee 0) \vee 0) \wedge (c(y) \vee c(x)) = c(x \vee 0)$	[12098 → 84,12927,12,12059]
13761	$x \vee ((0 \vee c((x \vee y) \vee 0)) \vee y) = 1$	[12524 → 250]
13959	$c(0 \vee c(x)) = x \wedge 1$	[199 → 12607,6383,5476,12035]
14074,14073	$(0 \vee x) \wedge 1 = x$	[12607 → 26,27,12059]
14119,14118	$x \vee (x \vee x) = x$	[13197:14074]
14142,14141	$x \wedge 1 = x$	[13191:14119]
14146,14145	$x \vee (y \wedge x) = x$	[12056:14119,14142]
14148,14147	$1 \wedge x = x$	[8359:14119]
14152,14151	$0 \vee x = x$	[4217:14119,14148]
14182,14181	$c(c(x)) = x$	[13959:14152,14142]
14228,14227	$x \vee x = x$	[13707:14142,14142]
14255	$x \vee c(c(x \vee y) \vee (z \vee y)) = x$	[13491:14228,14142]
14258,14257	$x \vee 0 = x$	[13483:14142,14142,14228,13644,14142]
14259	$x \vee ((x \vee c(y)) \wedge y) = x$	[13479:14228,14142]
14279	$(x \vee y) \wedge (x \vee c(y)) = x$	[13253:14142,14152]
14292,14291	$c(x \vee y) = c(x) \wedge c(y)$	[12958:14258,14258,14258,14142]
14293	$(x \vee c(y)) \wedge (y \vee x) = x$	[12566:14142,14258]
14295	$c(x) \vee (y \vee x) = 1$	[12518:14258,14142]
14299	$x \vee (c(y) \wedge (c(z) \wedge x)) = x$	[12490:14292,14292,14182,14142]
14309	$x \vee (x \wedge c(y)) = x$	[12288:14258,14258,14292,14182,14142]
14319	$x \vee (c(x) \vee y) = 1$	[12162:14142,14258]
14349	$(x \vee (c(y) \wedge z)) \wedge (y \vee (x \vee c(z))) = x$	[3123:14258,14142]
14351	$(x \vee (c(y) \vee c(z))) \wedge ((y \wedge z) \vee x) = x$	[665:14142,14258]
14353	$(x \vee y) \wedge (c(y) \vee x) = x$	[424:14142,14258]
14361	$(x \vee c(y)) \vee (c(x) \wedge y) = 1$	[234:14142,14258]
14366,14365	$c(x \wedge y) = c(x) \vee c(y)$	[84:14258,14142]
14369	$x \vee ((c(x) \wedge c(y)) \vee y) = 1$	[13761:14258,14292,14152]
14373	$c(x) \wedge (c(y) \vee c(x)) = c(x)$	[13751:14258,14258,14258]
14431	$x \vee ((x \vee y) \wedge c(y)) = x$	[13050:14228,14292,14152,14292,14366,14182,14182,14148]
14464	$x \vee ((x \vee y) \wedge (c(z) \wedge c(y))) = x$	[14255:14292,14292,14366,14182,14182,14292]
14527	$x \vee (y \vee x) = y \vee x$	[14227 → 2]
14562	$x \vee (y \vee z) = z \vee (y \vee x)$	[14257 → 98,14258]
14563	$x \vee (y \vee z) = y \vee (z \vee x)$	[14257 → 15,14258]
14565	$x \vee y = y \vee x$	[14257 → 2,14258]
14598	$x \vee (y \vee (z \wedge x)) = y \vee x$	[14145 → 2]
14601,14600	$(x \wedge y) \vee y = y$	[14145 → 14565]
14737	$(c(x) \vee y) \wedge (y \vee x) = y$	[14565 → 7]

14741	$(c(x) \vee c(y)) \wedge (y \wedge x) = 0$	[14565 → 36]
14829	$x \vee ((y \wedge (x \vee z)) \vee z) = x \vee z$	[2 → 14600]
14857	$c(x) \wedge ((y \wedge c(x)) \vee x) = y \wedge c(x)$	[14600 → 7]
14863	$(x \vee y) \vee c(y) = 1$	[14565 → 14295]
14867	$x \vee (x \wedge y) = x$	[14181 → 14309]
14880,14879	$(x \wedge y) \vee x = x$	[14565 → 14867]
14907	$x \vee (y \vee (x \wedge z)) = y \vee x$	[14867 → 2]
14965	$(c(x) \vee y) \vee x = 1$	[14565 → 14319]
15141	$(c(x) \vee y) \vee (z \vee x) = 1$	[14965 → 2,12035]
15281	$x \vee ((c(y) \vee x) \wedge y) = x$	[14565 → 14259]
15300	$x \vee ((c(x) \wedge y) \vee c(y)) = 1$	[14259 → 14863,14366,14292,14182]
15332	$(x \vee y) \wedge (y \vee c(x)) = y$	[14565 → 14279]
15549	$c(x) \wedge (x \vee (c(x) \wedge y)) = c(x) \wedge y$	[14879 → 14293]
15745	$((x \vee y) \wedge c(y)) \vee x = x$	[14565 → 14431]
16074	$(c(x) \vee y) \vee (x \vee z) = 1$	[14565 → 15141]
16201	$x \vee (y \wedge (c(z) \wedge x)) = x$	[14181 → 14299]
16851	$x \vee (y \wedge (z \wedge x)) = x$	[14181 → 16201]
16855	$(x \wedge (y \wedge z)) \vee z = z$	[14565 → 16851]
16911,16910	$(x \wedge y) \vee (y \vee z) = y \vee z$	[14737 → 16855]
17134	$x \vee ((x \wedge c(y)) \wedge z) = x$	[16074 → 14349,14292,14182,14142]
17188	$(x \wedge y) \vee (x \vee z) = x \vee z$	[16074 → 14351,14182,14148,14182]
17368	$x \vee ((x \wedge y) \wedge z) = x$	[14181 → 17134]
17488	$c(x) \vee (((x \wedge c(y)) \vee y) \wedge x) = 1$	[15745 → 14361,14366,14292,14182,14182]
17608	$((x \wedge c(y)) \vee y) \vee x = (x \wedge c(y)) \vee y$	[14369 → 15281,14182,14148,14182]
17694	$x \wedge (c(y) \vee x) = x$	[14181 → 14373,14182,14182]
17732	$x \wedge (y \vee x) = x$	[14181 → 17694]
17784	$(x \wedge y) \wedge y = x \wedge y$	[14145 → 17732]
18416,18415	$(x \vee y) \vee z = y \vee (x \vee z)$	[14565 → 14562]
18569,18568	$(x \wedge c(y)) \vee y = y \vee x$	[17608:18416,14880]
18926	$c(x) \vee ((y \vee x) \wedge x) = 1$	[17488:18569]
18940	$c(x) \wedge (x \vee y) = y \wedge c(x)$	[14857:18569]
18943	$x \wedge c(y) = c(y) \wedge (y \vee x)$	[18940]
18987	$x \vee (y \wedge c(x)) = x \vee y$	[17784 → 18568,18569]
19019	$x \wedge (c(x) \vee y) = y \wedge x$	[18568 → 15332,14182,14146,14182]
19036,19035	$(c(x) \vee y) \wedge x = y \wedge x$	[18568 → 14293,14182,14146,14182]
19045	$x \wedge y = y \wedge (c(y) \vee x)$	[19019]
19085	$(x \vee y) \wedge y = y$	[18926 → 14353,14601,14142]
19111	$(x \vee y) \wedge x = x$	[18568 → 19085]
19120,19119	$x \wedge (y \wedge (z \wedge x)) = y \wedge (z \wedge x)$	[16851 → 19085]
19255	$c(x) \vee (y \wedge x) = c(x) \vee y$	[14181 → 18987]
19297	$x \wedge (y \vee c(x)) = y \wedge x$	[14565 → 19019]
19301,19300	$(x \vee c(y)) \wedge y = y \wedge (x \vee c(y))$	[14527 → 19019]
19322	$x \wedge y = y \wedge (x \vee c(y))$	[19297]
19355	$x \vee (y \wedge (x \vee c(y))) = x$	[14259:19301]
19590	$(c(x) \wedge (x \vee y)) \vee x = x \vee y$	[19045 → 18568,14182]
19634	$(x \wedge (c(x) \vee y)) \vee y = y$	[19045 → 14879]
19852	$(x \wedge y) \vee (y \wedge (c(y) \vee x)) = x \wedge y$	[18568 → 19355,14182,14182]
20391,20390	$(x \vee (y \wedge z)) \wedge (x \vee z) = x \vee (y \wedge z)$	[14598 → 17732]
20402	$(x \vee (y \wedge c(z))) \wedge z = z \wedge (x \vee c(z))$	[14598 → 19035,19301]
20556	$(x \vee (c(y) \wedge z)) \wedge y = y \wedge (x \vee c(y))$	[14907 → 19035,19301]
20672	$x \vee ((x \wedge y) \vee c(y)) = (x \wedge y) \vee c(y)$	[15300 → 19634,14142,14182,14182]
20674	$((x \wedge y) \vee c(y)) \wedge x = x$	[15300 → 19035,14148,14182]
20678	$((x \wedge (y \vee c(x))) \vee c(x)) \wedge y = y$	[19322 → 20674]
20707,20706	$x \vee (c(x) \wedge y) = x \vee y$	[14279 → 20674,14292,14182,20391]

20713,20712	$c(x) \wedge (x \vee y) = c(x) \wedge y$	[15549:20707]
20718	$(c(x) \wedge y) \vee x = x \vee y$	[19590:20713]
20720	$x \wedge c(y) = c(y) \wedge x$	[18943:20713]
20740	$x \wedge y = y \wedge x$	[14181 → 20720,14182]
20742,20741	$c(x) \wedge (y \vee x) = c(x) \wedge y$	[19322 → 20720,14182]
20768,20767	$x \wedge (y \vee c(x)) = x \wedge y$	[19322 → 20740]
20770,20769	$x \wedge (c(x) \vee y) = x \wedge y$	[19045 → 20740]
20775	$((x \wedge y) \vee c(x)) \wedge y = y$	[20678:20768]
20778,20777	$(x \vee (c(y) \wedge z)) \wedge y = y \wedge x$	[20556:20768]
20780,20779	$(x \vee (y \wedge c(z))) \wedge z = z \wedge x$	[20402:20768]
20783	$(x \wedge y) \vee (y \wedge x) = x \wedge y$	[19852:20770]
20960,20959	$(x \vee c(y)) \wedge y = y \wedge x$	[20740 → 20767]
21072	$(x \wedge y) \wedge (y \wedge x) = y \wedge x$	[14741 → 20775,14292,14182,14182,14152]
21087	$x \vee ((y \wedge x) \vee c(y)) = (y \wedge x) \vee c(y)$	[20775 → 14879]
21465,21464	$x \vee ((y \wedge x) \vee z) = x \vee z$	[2 → 16910]
21469,21468	$(x \wedge y) \vee c(x) = y \vee c(x)$	[21087:21465]
21588,21587	$x \vee ((x \wedge y) \vee z) = x \vee z$	[2 → 17188]
21594,21593	$(x \wedge y) \vee c(y) = x \vee c(y)$	[20672:21588]
22207	$(x \wedge c(y)) \vee (x \wedge y) = x$	[20769 → 19255,14292,14182,14292,14182,14880]
22209	$(c(x) \wedge y) \vee (y \wedge x) = y$	[20767 → 19255,14292,14182,14292,14182,14601]
22240	$(x \wedge c(y)) \vee (y \wedge x) = x$	[20740 → 22207]
22267	$(x \wedge y) \vee (y \wedge c(x)) = y$	[14181 → 22209]
22271	$(x \wedge y) \vee (c(y) \wedge x) = x$	[14565 → 22209]
22394	$(x \wedge y) \vee (c(x) \wedge y) = y$	[20740 → 22267]
22462	$x \vee ((x \vee y) \wedge (z \wedge c(y))) = x$	[14181 → 14464]
23045,23044	$(x \wedge y) \vee ((y \wedge x) \vee z) = (y \wedge x) \vee z$	[20783 → 18415]
23048	$x \vee (y \wedge z) = (z \wedge y) \vee x$	[20783 → 14563,23045]
23273	$(x \wedge y) \vee ((y \wedge x) \wedge z) = x \wedge y$	[21072 → 17368]
23304,23303	$(x \vee (y \vee c(z))) \wedge (z \vee x) = (y \wedge z) \vee x$	[21464 → 14293,18416,21594]
23439,23438	$((c(x) \wedge y) \vee z) \wedge x = z \wedge x$	[21587 → 19035,19036]
24841	$x \wedge (y \wedge (x \vee c(z))) = y \wedge x$	[18568 → 20777,23439,14366,14182]
25350	$x \vee ((y \vee x) \wedge (z \wedge c(y))) = x$	[14565 → 22462]
26809,26808	$x \wedge (y \wedge (x \vee z)) = y \wedge x$	[14181 → 24841]
26840	$(x \wedge y) \wedge (z \wedge x) = z \wedge (x \wedge y)$	[22271 → 26808]
26851	$x \wedge (y \wedge (z \vee x)) = y \wedge x$	[14907 → 26808]
26869	$x \wedge ((x \vee y) \wedge z) = x \wedge z$	[20959 → 26808,14292,20778]
26872	$(x \wedge (y \vee z)) \wedge y = x \wedge y$	[17784 → 26808,26809]
26983,26982	$(x \wedge (y \vee z)) \vee c(y) = x \vee c(y)$	[26808 → 21468,21594]
27026	$x \wedge ((y \vee x) \wedge z) = x \wedge z$	[20959 → 26851,14292,20780]
27096	$x \vee (y \wedge (z \vee c(x))) = x \vee y$	[26851 → 20718,18569]
27140	$(x \wedge (y \vee z)) \vee c(z) = x \vee c(z)$	[26851 → 21468,21594]
27178,27177	$(x \wedge y) \wedge (y \wedge z) = (x \wedge y) \wedge z$	[22394 → 26869]
27266	$(((x \vee y) \wedge z) \wedge c(x)) \vee (x \wedge z) = (x \vee y) \wedge z$	[26869 → 22240]
27342,27341	$(x \wedge y) \wedge z = x \wedge (y \wedge z)$	[22271 → 26872,27178]
27373	$((x \vee y) \wedge (z \wedge c(x))) \vee (x \wedge z) = (x \vee y) \wedge z$	[27266:27342]
27452	$x \wedge (y \wedge z) = y \wedge (z \wedge x)$	[26840:27342,19120]
27776	$(x \wedge y) \vee (y \wedge (x \wedge z)) = x \wedge y$	[23273:27342]
28022,28021	$((x \vee y) \wedge z) \vee c(y) = z \vee c(y)$	[27026 → 21468,21469]
28503,28502	$(x \vee y) \wedge (z \wedge c(x)) = y \wedge (z \wedge c(x))$	[25350 → 14293,28022,20960]
28505	$(x \wedge (y \wedge c(z))) \vee (z \wedge y) = (z \vee x) \wedge y$	[27373:28503]
28507	$(x \vee y) \wedge z = (y \wedge (z \wedge c(x))) \vee (x \wedge z)$	[28505]
28888	$((x \vee c(y)) \wedge z) \vee y = y \vee z$	[23048 → 27096]
28890	$(x \wedge (y \vee c(z))) \vee z = z \vee x$	[14565 → 27096]
30272,30271	$(x \wedge (y \wedge (z \vee c(u)))) \vee u = u \vee (x \wedge y)$	[27452 → 28888]

30647	$(x \wedge (y \vee z)) \vee z = (x \wedge y) \vee z$	[14829 → 14293,18416,26983,23304]
32813	$(x \vee c(y)) \wedge (y \vee (x \wedge (z \vee y))) = x \wedge (z \vee y)$	[27140 → 14353,14182]
33040	$(x \wedge ((x \wedge y) \vee z)) \vee (x \wedge y) = x \wedge ((x \wedge y) \vee z)$	[19111 → 27776]
33573,33572	$(x \wedge (y \vee z)) \vee y = y \vee (x \wedge z)$	[28890 → 30647,30272]
33587,33586	$(x \wedge (y \wedge c(z))) \vee (z \wedge y) = (x \wedge y) \vee (z \wedge y)$	[22240 → 30647]
33624,33623	$x \vee (y \wedge (z \vee x)) = (y \wedge z) \vee x$	[14565 → 30647]
33638,33637	$x \wedge ((x \wedge y) \vee z) = (x \wedge y) \vee (x \wedge z)$	[33040:33573]
33646,33645	$(x \vee y) \wedge z = (y \wedge z) \vee (x \wedge z)$	[28507:33587]
33672	$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$	[32813:33624,33646,20742,33638,16911]
33674	$\$Ans(DIST1)$	[33672,13]

Proof BA2

2	$x \vee (y \vee z) = y \vee (x \vee z)$	[]
3	$x \wedge y = c(c(x) \vee c(y))$	[]
4	$c(c(x) \vee c(y)) = x \wedge y$	[3]
7	$(x \vee c(y)) \wedge (x \vee y) = x$	[]
10,9	$x \vee c(x) = 1$	[]
12,11	$c(1) = 0$	[]
13	$A \vee (B \wedge (A \vee C)) \neq A \vee (C \wedge (A \vee B)) \mid \$Ans(MOD)$	[]
14	$A \vee (C \wedge (A \vee B)) \neq A \vee (B \wedge (A \vee C)) \mid \$Ans(MOD)$	[13]
15	$x \vee (y \vee (z \vee u)) = z \vee (x \vee (y \vee u))$	[2 → 2]
16	$x \vee (y \vee (z \vee u)) = y \vee (z \vee (x \vee u))$	[15]
18,17	$1 \vee 0 = 1$	[11 → 9]
19	$x \vee (y \vee c(x)) = y \vee 1$	[9 → 2]
21	$x \vee 1 = 1 \vee (x \vee 0)$	[17 → 2]
22	$1 \vee (x \vee 0) = x \vee 1$	[21]
23	$c(0 \vee c(x)) = 1 \wedge x$	[11 → 4]
25	$c((x \wedge y) \vee c(z)) = (c(x) \vee c(y)) \wedge z$	[4 → 4]
28,27	$c(c(x) \vee 0) = x \wedge 1$	[11 → 4]
30,29	$c(c(x) \vee (y \wedge z)) = x \wedge (c(y) \vee c(z))$	[4 → 4]
31	$x \wedge c(x) = 0$	[9 → 4,12]
33	$(c(x) \vee c(y)) \vee (x \wedge y) = 1$	[4 → 9]
35	$1 \wedge 0 = 0$	[11 → 31]
37	$(c(x) \vee c(y)) \wedge (x \wedge y) = 0$	[4 → 31]
39	$x \vee (1 \vee (y \vee 0)) = y \vee (x \vee 1)$	[21 → 2]
40	$x \vee (y \vee 1) = y \vee (1 \vee (x \vee 0))$	[39]
42,41	$(x \vee 0) \wedge (x \vee 1) = x$	[11 → 7]
43	$(x \vee (y \wedge z)) \wedge (x \vee (c(y) \vee c(z))) = x$	[4 → 7]
46,45	$1 \wedge (x \vee x) = x$	[9 → 7]
47	$(x \vee 0) \wedge (1 \vee (x \vee 0)) = x$	[21 → 7,12]
49	$(1 \vee c(0)) \wedge 1 = 1$	[17 → 7]
51	$(x \vee c(c(x))) \wedge 1 = x$	[9 → 7]
53	$(x \vee c(y \vee z)) \wedge (y \vee (x \vee z)) = x$	[2 → 7]
56,55	$1 \wedge (x \vee ((x \vee y) \vee y)) = x \vee y$	[2 → 45]
59	$x \vee (y \vee 1) = 1 \vee (x \vee (y \vee 0))$	[22 → 2]
60	$1 \vee (x \vee (y \vee 0)) = x \vee (y \vee 1)$	[59]
64,63	$c(0 \vee 0) = 1 \wedge 1$	[11 → 23]
65	$c(0 \vee (x \wedge y)) = 1 \wedge (c(x) \vee c(y))$	[4 → 23]
67	$(x \vee (1 \wedge y)) \wedge (x \vee (0 \vee c(y))) = x$	[23 → 7]
71	$(0 \vee c(x)) \vee (1 \wedge x) = 1$	[23 → 9]
73	$(x \vee (1 \wedge 1)) \wedge (x \vee (0 \vee 0)) = x$	[63 → 7]

86,85	$c((x \wedge y) \vee 0) = (c(x) \vee c(y)) \wedge 1$	[4 → 27]
91	$(c(x) \vee 0) \vee (x \wedge 1) = 1$	[27 → 9]
95	$x \vee (y \vee (z \vee c(x))) = y \vee (z \vee 1)$	[9 → 15]
99	$x \vee (y \vee (z \vee u)) = z \vee (y \vee (x \vee u))$	[2 → 15]
106	$(x \vee c(y \vee (z \vee u))) \wedge (z \vee (x \vee (y \vee u))) = x$	[15 → 7]
128	$1 \wedge (x \vee (y \vee ((x \vee (y \vee z)) \vee z))) = x \vee (y \vee z)$	[16 → 45]
131	$(x \vee c(y \vee (z \vee u))) \wedge (y \vee (z \vee (x \vee u))) = x$	[16 → 7]
138	$(c(x) \vee 0) \vee (y \vee (x \wedge 1)) = y \vee 1$	[27 → 19]
142	$(c(x) \vee c(y)) \vee (z \vee (x \wedge y)) = z \vee 1$	[4 → 19]
162,161	$c(x \vee c(y)) = (c(x \vee 0) \vee c(x \vee 1)) \wedge y$	[41 → 25]
164,163	$((1 \wedge 1) \vee c(0 \vee 1)) \wedge x = 1 \wedge x$	[35 → 25,162,64,12,10]
175	$(0 \vee (1 \wedge 1)) \vee 0 = 1$	[63 → 71,46]
179	$(0 \vee c(x \vee x)) \vee x = 1$	[45 → 71]
183	$1 \wedge ((0 \vee (1 \wedge 1)) \vee 1) = 0 \vee (1 \wedge 1)$	[175 → 41]
186,185	$(0 \vee (1 \wedge 1)) \vee 1 = 1 \vee 1$	[175 → 22]
192	$0 \vee (1 \wedge 1) = 1$	[183:186,46]
197,196	$c((x \wedge 1) \vee (y \wedge z)) = (c(x) \vee 0) \wedge (c(y) \vee c(z))$	[27 → 29]
198	$c(c(x) \vee 1) = x \wedge (((0 \vee c(1 \vee 1)) \wedge 0) \vee 0)$	[49 → 29,162,18,12,12]
200	$c(c(x) \vee y) = x \wedge (0 \vee c(y \vee y))$	[45 → 29,12]
201	$c(c(x) \vee y) = x \wedge (c(y \vee 0) \vee c(y \vee 1))$	[41 → 29]
202	$x \wedge (0 \vee c(y \vee y)) = c(c(x) \vee y)$	[200]
203	$x \wedge (c(y \vee 0) \vee c(y \vee 1)) = c(c(x) \vee y)$	[201]
206	$(c(x) \vee (y \wedge z)) \vee (x \wedge (c(y) \vee c(z))) = 1$	[29 → 9]
211	$(0 \vee c(1 \wedge 1)) \wedge 1 = 0$	[192 → 7]
213	$0 \vee (x \vee (1 \wedge 1)) = x \vee 1$	[192 → 2]
227	$(((c(x \vee 0) \vee c(x \vee 1)) \wedge c(x)) \vee 0) \vee x = 1$	[51 → 91,162]
235	$((x \wedge 1) \vee c(y)) \vee ((c(x) \vee 0) \wedge y) = 1$	[27 → 33]
241	$(c(x \vee 0) \vee c(x \vee 1)) \vee x = 1$	[41 → 33]
251	$x \vee ((0 \vee c((x \vee y) \vee (x \vee y))) \vee y) = 1$	[2 → 179]
256	$x \vee (y \vee 1) = (0 \vee c(z \vee z)) \vee (x \vee (y \vee z))$	[179 → 15]
259	$x \vee 1 = (0 \vee c(y \vee y)) \vee (x \vee y)$	[179 → 2]
261	$(0 \vee c(x \vee x)) \vee (y \vee (z \vee x)) = y \vee (z \vee 1)$	[256]
264,263	$((1 \wedge (1 \wedge 1)) \vee 0) \vee 0 = 1$	[211 → 91,162,64,164]
279	$c(c(x) \vee y) = x \wedge (c(y \vee 0) \vee c(1 \vee (y \vee 0)))$	[47 → 29]
288,287	$(0 \vee 0) \vee 1 = 0 \vee 1$	[91 → 213,12]
307	$(x \vee c(1 \vee (y \vee 0))) \wedge (y \vee (x \vee 1)) = x$	[39 → 7]
313,312	$(1 \wedge (1 \wedge 1)) \vee 0 = 1$	[263 → 47,264,46]
315	$1 \wedge (1 \wedge 1) = 1$	[312 → 47,313,46]
323	$x \vee (y \vee 1) = x \vee (1 \vee (y \vee 0))$	[2 → 40]
326	$x \vee (1 \vee (y \vee 0)) = x \vee (y \vee 1)$	[323]
335	$(x \vee c(y \vee 1)) \wedge (y \vee (1 \vee (x \vee 0))) = x$	[40 → 7]
377	$(x \vee y) \wedge (x \vee (0 \vee c(y \vee y))) = x$	[45 → 43,12]
381	$1 \wedge (0 \vee (0 \vee 0)) = 0$	[192 → 43,12,12]
389	$(x \vee (y \wedge (c(z) \vee 0))) \wedge (x \vee (c(y) \vee (z \wedge 1))) = x$	[27 → 43]
391	$(x \vee (y \wedge x)) \wedge (c(y) \vee 1) = x$	[19 → 43]
393	$(x \vee (y \wedge z)) \wedge (c(y) \vee (x \vee c(z))) = x$	[2 → 43]
400,399	$(0 \vee c(0 \vee (0 \vee 0))) \vee 0 = 1$	[381 → 71]
423	$(x \vee (1 \wedge 1)) \wedge (0 \vee (x \vee 0)) = x$	[63 → 53]
425	$(x \vee (y \wedge 1)) \wedge (c(y) \vee (x \vee 0)) = x$	[27 → 53]
441	$(x \vee c((0 \vee c((x \vee y) \vee (x \vee y))) \vee y)) \wedge 1 = x$	[179 → 53]
445	$(1 \vee c(x \vee (y \vee 0))) \wedge (y \vee (x \vee 1)) = 1$	[39 → 53]
449	$(x \vee c(y \vee (z \vee u))) \wedge (z \vee (y \vee (x \vee u))) = x$	[15 → 53]
467	$((x \vee x) \vee x) \wedge (0 \vee 1) = x \vee x$	[45 → 391,12]
501	$1 \wedge (x \vee (y \vee ((y \vee (x \vee z)) \vee z))) = y \vee (x \vee z)$	[15 → 55]

506	$0 \vee c(0 \vee (0 \vee 0)) = 1$	[399 → 47,400,46]
521	$0 \vee (x \vee c(0 \vee (0 \vee 0))) = x \vee 1$	[506 → 2]
562,561	$(0 \vee c((x \vee 0) \vee (x \vee 0))) \vee (x \vee 1) = 1 \vee 1$	[179 → 60]
581	$1 \wedge (c(x \vee 0) \vee c(1 \vee (x \vee 0))) = c(0 \vee x)$	[47 → 65]
583	$c(0 \vee x) = 1 \wedge (0 \vee c(x \vee x))$	[45 → 65,12]
585	$1 \wedge (0 \vee c(x \vee x)) = c(0 \vee x)$	[583]
619	$(x \vee (1 \wedge y)) \wedge (0 \vee (x \vee c(y))) = x$	[2 → 67]
632,631	$((1 \wedge 1) \vee c(0 \vee 1)) \vee 0 = 1$	[63 → 241]
644	$(1 \wedge 1) \vee c(0 \vee 1) = 1$	[241 → 47,64,632,46,64]
659	$c(x \vee 0) = (0 \vee c(x \vee x)) \wedge 1$	[45 → 85,12]
663	$(0 \vee c(x \vee x)) \wedge 1 = c(x \vee 0)$	[659]
666	$(x \vee ((c(y) \vee c(z)) \wedge 1)) \wedge ((y \wedge z) \vee (x \vee 0)) = x$	[85 → 53]
670	$((x \wedge y) \vee 0) \wedge ((c(x) \vee c(y)) \wedge 1) = 0$	[85 → 31]
684	$(1 \wedge 1) \vee c(1 \vee (0 \vee 0)) = 1$	[21 → 644]
686	$1 \wedge (0 \vee ((1 \wedge 1) \vee 1)) = 1 \wedge 1$	[644 → 53]
689,688	$(1 \wedge 1) \vee 1 = (0 \vee 1) \vee 1$	[644 → 19]
704,703	$1 \wedge 1 = 0 \vee 1$	[686:689,56]
711	$(0 \vee 1) \vee c(1 \vee (0 \vee 0)) = 1$	[684:704]
733	$(x \vee (0 \vee 1)) \wedge (0 \vee (x \vee 0)) = x$	[423:704]
756,755	$1 \wedge (0 \vee 1) = 1$	[315:704]
773	$0 \vee (x \vee (0 \vee 1)) = x \vee 1$	[213:704]
779	$0 \vee (0 \vee 1) = 1$	[192:704]
789	$(x \vee (0 \vee 1)) \wedge (x \vee (0 \vee 0)) = x$	[73:704]
792,791	$c(0 \vee 0) = 0 \vee 1$	[63:704]
793	$(1 \vee (0 \vee 1)) \wedge (0 \vee 1) = 1$	[703 → 391,12]
795	$c(c(x) \vee (0 \vee 1)) = x \wedge (0 \vee 0)$	[703 → 29,12,12]
807	$0 \vee (1 \vee (0 \vee 0)) = 1$	[21 → 779]
812,811	$0 \vee (0 \vee (x \vee 1)) = x \vee 1$	[779 → 16]
815	$c((0 \vee 1) \vee (x \wedge y)) = (0 \vee 0) \wedge (c(x) \vee c(y))$	[791 → 29]
841	$0 \vee (1 \vee (x \vee (0 \vee 0))) = x \vee 1$	[807 → 16]
861	$0 \vee (0 \vee (1 \vee (x \vee 0))) = x \vee 1$	[40 → 773]
863	$x \vee 1 = 0 \vee (0 \vee (1 \vee (x \vee 0)))$	[861]
867,866	$(0 \vee (1 \vee 1)) \wedge (0 \vee 1) = 1$	[40 → 793,18]
926,925	$(1 \vee (0 \vee 0)) \vee 1 = (0 \vee 1) \vee 1$	[711 → 19]
1074,1073	$1 \wedge (0 \vee c((x \vee y) \vee (x \vee y))) = c(x \vee (0 \vee y))$	[2 → 583]
1188	$(0 \vee (x \vee 1)) \wedge (0 \vee (x \vee 0)) = x$	[2 → 733]
1212	$(x \vee c(y \vee (z \vee c(x)))) \wedge (z \vee (y \vee 1)) = x$	[19 → 106]
1230	$(0 \vee (x \vee 1)) \wedge (x \vee (0 \vee 0)) = x$	[2 → 789]
1233,1232	$0 \vee c((0 \vee 0) \vee (0 \vee 0)) = (1 \vee 1) \wedge 1$	[179 → 789,562]
1295,1294	$(c((0 \vee 0) \vee 0) \vee c(0 \vee 1)) \vee 1 = 0 \vee (1 \vee 1)$	[241 → 841,288]
1297,1296	$((1 \vee 1) \wedge 1) \vee 1 = 0 \vee (1 \vee 1)$	[179 → 841,1233]
1319	$c(x \vee 1) = c(0 \vee (0 \vee (1 \vee (x \vee 0))))$	[861 → 583,1074]
1326,1325	$0 \vee (0 \vee (1 \vee (x \vee 0))) = 1 \vee (x \vee 0)$	[21 → 863]
1328	$c(x \vee 1) = c(1 \vee (x \vee 0))$	[1319:1326]
1441	$c((0 \vee 0) \vee 0) \vee c(0 \vee 1) = (1 \vee 1) \wedge 1$	[241 → 1230,288,1295,812,288]
1590,1589	$c(1 \vee (c(x) \vee 0)) = x \wedge (0 \vee c(1 \vee 1))$	[21 → 200]
1625	$c((1 \wedge c(x \vee x)) \vee x) = 0$	[7 → 202,162,792,10]
1684	$((1 \wedge c(x \vee x)) \vee x) \vee 0 = 1$	[1625 → 9]
1705,1704	$x \vee (((1 \wedge c(y \vee y)) \vee y) \vee 1) = x \vee (1 \vee 1)$	[1684 → 326]
1706	$(1 \wedge c(x \vee x)) \vee x = 1$	[1684 → 1188,1705,867]
1756	$x \vee 1 = (1 \wedge c(y \vee y)) \vee (x \vee y)$	[1706 → 2]
1763	$(1 \wedge c(x \vee x)) \vee (y \vee x) = y \vee 1$	[1756]
1868	$(x \vee y) \wedge (0 \vee (x \vee c(y \vee y))) = x$	[2 → 377]
1878	$(x \vee ((c(y) \vee 0) \wedge 1)) \wedge ((y \wedge 1) \vee (x \vee 0)) = x$	[27 → 425]

1912	$(0 \vee (0 \vee 0)) \vee (x \vee 1) = 0 \vee (x \vee 1)$	[521 → 95]
2368	$(c(x) \vee (1 \wedge y)) \vee (x \wedge (0 \vee c(y))) = 1$	[11 → 206]
2370	$(0 \vee (x \wedge x)) \vee c(x) = 1$	[45 → 206,12]
2379,2378	$(0 \vee (x \wedge x)) \vee 1 = x \vee 1$	[2370 → 19]
2397	$(0 \vee (x \wedge x)) \vee (y \vee c(x)) = y \vee 1$	[2370 → 2]
2404,2403	$1 \vee ((0 \vee (x \wedge x)) \vee 0) = x \vee 1$	[21 → 2378]
2463,2462	$x \vee (((1 \wedge c(0)) \vee 0) \vee 1) = x \vee (1 \vee 1)$	[227 → 326,792,10]
2464	$(1 \wedge c(0)) \vee 0 = 1$	[227 → 1188,792,10,2463,867,792,10]
2469,2468	$(1 \wedge c(0)) \vee 1 = 1 \vee 1$	[2464 → 1763,792,756]
2480	$1 \wedge c(0) = 1$	[2464 → 1188,2469,867]
2485,2484	$(c(0) \vee 1) \wedge (0 \vee 1) = c(0)$	[2480 → 391,12]
2486	$(x \vee 1) \wedge (0 \vee (x \vee c(c(0)))) = x$	[2480 → 619]
2533	$(1 \vee (c(0) \vee 0)) \wedge (0 \vee 1) = c(0)$	[21 → 2484]
2565	$0 \vee (c(0) \wedge c(0)) = c(0)$	[2370 → 2486,2379,2485]
2584	$(0 \vee c(c(0) \wedge c(0))) \wedge c(0) = 0$	[2565 → 7]
2606	$((0 \wedge (0 \vee c(1 \vee 1))) \vee c(0 \vee 1)) \wedge 1 = 0 \wedge 1$	[2533 → 85,28,1590]
2614	$((0 \wedge (0 \vee c(1 \vee 1))) \vee c(0 \vee 1)) \vee c(0) = 1$	[2533 → 33,1590]
3124	$(x \vee ((c(y) \vee 0) \wedge z)) \wedge ((y \wedge 1) \vee (x \vee c(z))) = x$	[27 → 393]
3345	$0 \vee ((x \vee 1) \wedge (x \vee 1)) = x \vee 1$	[2370 → 335,2404,56]
3354,3353	$(c(x \vee 1) \vee c(x \vee 1)) \vee (x \vee 1) = 0 \vee 1$	[3345 → 142]
3542	$(0 \vee (0 \vee 0)) \vee (0 \vee (1 \vee 1)) = 1 \vee 1$	[1296 → 1912,1297,812]
3896	$((1 \wedge c(x)) \vee (1 \wedge x)) \vee 0 = 1$	[7 → 2368,162,792,10]
3925,3924	$((1 \wedge c(x)) \vee (1 \wedge x)) \vee 1 = 1 \vee 1$	[3896 → 1763,792,756]
3929	$(1 \wedge c(x)) \vee (1 \wedge x) = 1$	[3896 → 1188,3925,867]
3965	$(1 \wedge x) \vee c(0 \vee x) = 1$	[585 → 3929,162,792,10,46]
4034	$x \vee c(0 \vee (x \vee x)) = 1$	[45 → 3965]
4113,4112	$(1 \wedge x) \vee 1 = (0 \vee x) \vee 1$	[3965 → 2397,2379]
4205	$x \vee c(x \vee (0 \vee x)) = 1$	[2 → 4034]
4219,4218	$1 \wedge (0 \vee (x \vee (x \vee x))) = x$	[4034 → 131]
4220	$1 \wedge (x \vee (x \vee (0 \vee x))) = x$	[4034 → 106]
4229	$(0 \vee (x \vee x)) \vee 1 = x \vee 1$	[4034 → 2397,2379]
4301,4300	$(x \vee (0 \vee x)) \vee 1 = x \vee 1$	[4205 → 2397,2379]
4438,4437	$1 \wedge (c(0) \vee (c(0) \vee 1)) = c(0)$	[95 → 4218]
4475	$1 \wedge ((1 \vee (0 \vee 0)) \vee ((0 \vee 1) \vee 1)) = 1 \vee (0 \vee 0)$	[807 → 4220,926]
4555	$(c(0) \vee 1) \vee 1 = c(0) \vee 1$	[19 → 4229]
4568,4567	$0 \vee (1 \vee 1) = 1$	[4229 → 128,4219]
4591,4590	$1 \vee 1 = 0 \vee 1$	[3542:4568,4301]
4879	$((0 \wedge (0 \vee c(0 \vee 1))) \vee c(0 \vee 1)) \vee c(0) = 1$	[2614:4591]
4885	$((0 \wedge (0 \vee c(0 \vee 1))) \vee c(0 \vee 1)) \wedge 1 = 0 \wedge 1$	[2606:4591]
4946,4945	$c(1 \vee (c(x) \vee 0)) = x \wedge (0 \vee c(0 \vee 1))$	[1589:4591]
4949	$c((0 \vee 0) \vee 0) \vee c(0 \vee 1) = (0 \vee 1) \wedge 1$	[1441:4591]
4977	$c(c(x) \vee 1) = x \wedge (((0 \vee c(0 \vee 1)) \wedge 0) \vee 0)$	[198:4591]
4983	$(0 \vee c(x \vee x)) \vee (1 \vee x) = 0 \vee 1$	[259 → 4590]
5311,5310	$c(0) \vee 1 = c(0)$	[4555 → 55,4438]
5348	$c(c(0)) = 0 \wedge (0 \vee c(0 \vee 1))$	[4555 → 1328,5311,5311,4946]
5359,5358	$c(0) = 0 \vee 1$	[4555 → 811,5311,10,5311,5311]
5407,5406	$0 \wedge (0 \vee c(0 \vee 1)) = c(0 \vee 1)$	[5348:5359]
5425,5424	$0 \vee 1 = 1$	[4879:5407,5359,3354]
5441,5440	$(0 \vee 0) \wedge 1 = 0$	[2584:5359,5425,5359,5425,704,5425,12,5359,5425]
5449	$((0 \wedge (0 \vee 0)) \vee 0) \wedge 1 = 0 \wedge 1$	[4885:5425,12,5425,12]
5457,5456	$1 \vee (0 \vee 0) = 1$	[4475:5425,4591,5425,926,5425,4591,5425,704,5425]
5467,5466	$0 \wedge (0 \vee 0) = 0$	[5406:5425,12,5425,12]
5477,5476	$c(0) = 1$	[5358:5425]
5500	$(0 \vee c(x \vee x)) \vee (1 \vee x) = 1$	[4983:5425]

5505,5504	$c(c(x) \vee 1) = x \wedge (((0 \vee 0) \wedge 0) \vee 0)$	[4977:5425,12]
5532	$c((0 \vee 0) \vee 0) \vee 0 = 1$	[4949:5425,12,5425,704,5425]
5536	$c(1 \vee (c(x) \vee 0)) = x \wedge (0 \vee 0)$	[4945:5425,12]
5653,5652	$1 \vee 1 = 1$	[4590:5425]
5839,5838	$c(1 \vee (x \wedge y)) = (0 \vee 0) \wedge (c(x) \vee c(y))$	[815:5425]
5841,5840	$x \wedge (((0 \vee 0) \wedge 0) \vee 0) = x \wedge (0 \vee 0)$	[795:5425,5505]
5843,5842	$c(0 \vee 0) = 1$	[791:5425]
5863,5862	$1 \wedge 1 = 1$	[703:5425]
5875,5874	$((x \vee x) \vee x) \wedge 1 = x \vee x$	[467:5425]
5914,5913	$0 \wedge 1 = 0$	[5449:5467,5441]
5949	$c(c(x) \vee 1) = x \wedge (0 \vee 0)$	[5504:5841]
5967	$(x \vee (0 \wedge x)) \wedge 1 = x$	[5476 → 391,5653]
6032,6031	$(0 \vee 0) \wedge (c(x) \vee c(y)) = 0 \wedge (c(x) \vee c(y))$	[5476 → 29,5839]
6054,6053	$c(1 \vee (x \wedge y)) = 0 \wedge (c(x) \vee c(y))$	[5838:6032]
6258,6257	$1 \wedge (c(x \vee (0 \wedge x)) \vee 0) = c(0 \vee x)$	[5967 → 65,12]
6384,6383	$0 \vee 0 = 0$	[5532 → 279,12,5843,5457,12,18,5875]
6392,6391	$c(c(x) \vee 1) = x \wedge 0$	[5949:6384]
6438,6437	$c(1 \vee (c(x) \vee 0)) = x \wedge 0$	[5536:6384]
6500,6499	$0 \vee (x \vee 0) = x \vee 0$	[6383 → 2]
6639	$(1 \wedge (x \vee 0)) \vee c(x \vee 0) = 1$	[6499 → 3965]
6783	$c(1 \vee ((x \wedge 0) \vee 0)) = (1 \vee (c(x) \vee 0)) \wedge 0$	[6437 → 6437]
6785	$(1 \vee (x \wedge 0)) \wedge (c(x) \vee 1) = 1$	[6437 → 445,5653]
7015	$(0 \vee (((x \wedge 1) \vee (x \wedge 0)) \wedge x)) \vee 1 = x \vee 1$	[5500 → 95,162,28,6392,5653]
7284	$1 \wedge ((x \wedge 1) \vee (x \wedge 0)) = 1 \wedge x$	[279 → 581,6384,5477,6384,18,12,18,6438,162,6384,5477,5425,12,18]
7346	$(1 \wedge (c(x) \vee 0)) \vee (x \wedge 1) = 1$	[279 → 6639,6384,5477,6384,18,12,18]
7392	$(x \vee 1) \wedge (x \vee ((0 \wedge (c(y) \vee 1)) \vee (y \wedge 0))) = x$	[6785 → 43,6054,5477,6392]
7680	$c(0 \vee x) \vee x = 1$	[5967 → 7346,6258]
7694	$c(x \vee 0) \vee (x \vee 0) = 1$	[6499 → 7680]
7744	$(1 \wedge x) \vee c(x) = 1$	[161 → 7680,6384,5477,5425,12,18]
7809,7808	$(x \vee x) \wedge 1 = x$	[7680 → 1868,162,6384,5477,5425,12,18,46,5425,162,6384,5477,5425,12,18,46]
7825	$(0 \vee c(x \vee x)) \vee 1 = c(0 \vee (y \vee x)) \vee (y \vee 1)$	[7680 → 261]
7921	$c(x \vee 0) \vee (x \vee 1) = 1$	[7680 → 60,5653,6500]
7950,7949	$(0 \vee x) \vee 1 = x \vee 1$	[7680 → 2397,2379,162,6384,5477,5425,12,18,4113]
8056	$c(x \vee x) \vee 1 = c(0 \vee (y \vee x)) \vee (y \vee 1)$	[7825:7950]
8076,8075	$(((x \wedge 1) \vee (x \wedge 0)) \wedge x) \vee 1 = x \vee 1$	[7015:7950]
8137	$(x \vee x) \vee 1 = x \vee 1$	[4229:7950]
8142,8141	$(1 \wedge x) \vee 1 = x \vee 1$	[4112:7950]
8153	$(x \wedge x) \vee 1 = x \vee 1$	[2378:7950]
8208	$(1 \wedge (c(x \vee x) \vee 0)) \vee x = 1$	[7808 → 7346]
8251	$c(x \vee 0) = (c(x \vee x) \vee 0) \wedge 1$	[7808 → 85,12]
8261	$(c(x \vee x) \vee 0) \wedge 1 = c(x \vee 0)$	[8251]
8289	$x \vee c(x \vee x) = 1$	[45 → 7744]
8350	$x \wedge (c(x) \vee c(x \vee 1)) = 0$	[8289 → 201,12,162,28,6392,86,197,5477,42,7809,162,28,6392,8076]
8354,8353	$1 \wedge (x \vee ((x \vee y) \vee ((x \vee y) \vee y))) = x \vee y$	[8289 → 449]
8360	$1 \wedge (x \vee (x \vee x)) = x$	[8289 → 53]
8566,8565	$(x \vee 1) \vee (x \vee 1) = x \vee 1$	[8137 → 501,8354]
8728,8727	$x \wedge (c(y) \vee c(y \vee 1)) = x \wedge (c(y) \vee c(y))$	[8153 → 203,86,7809,30]
8780	$x \wedge (c(x) \vee c(x)) = 0$	[8350:8728]
8822	$x \vee c(x \vee (x \vee x)) = 1$	[8360 → 7744]
8852	$(c(x) \vee 1) \wedge ((x \wedge 0) \vee (x \wedge 0)) = 0$	[6391 → 8780,6392]
8878	$(x \wedge 1) \vee (c(x) \vee 0) = 1$	[279 → 7694,6384,5477,6384,18,12,18]
8894	$(x \vee c(c(x \vee 0) \vee (y \vee 0))) \wedge (y \vee 1) = x$	[7694 → 449]

9086	$(x \wedge 1) \vee (c(x) \vee 1) = 1$	[279 → 7921,6384,5477,6384,18,12,18]
9090	$c(x) \vee (x \vee 1) = 1$	[8153 → 7921,86,7809]
9111,9110	$c(x \vee 1) \vee 1 = c(x \vee 0) \vee 1$	[7921 → 1763,8566,8142]
9200	$c(0 \vee x) \vee (x \vee 1) = 1$	[7949 → 9090]
9222	$x \vee (c(x) \vee 1) = 1$	[2 → 9090]
9226	$(x \vee (x \wedge 0)) \wedge 1 = x$	[9090 → 307,6438]
9229,9228	$c(x \vee 0) \vee 1 = c(x) \vee 1$	[9090 → 1763,8566,8142,9111]
9381,9380	$(x \wedge 0) \vee 1 = x \vee 1$	[9222 → 1763,8566,6392,8142]
9398	$(x \vee c(y \vee (z \vee (c(x) \vee 1)))) \wedge (y \vee (z \vee 1)) = x$	[9222 → 131]
9496	$1 \wedge (c(x \vee (x \wedge 0)) \vee 0) = c(0 \vee x)$	[9226 → 65,12]
9517,9516	$1 \vee ((x \wedge 0) \vee 0) = x \vee 1$	[21 → 9380]
9519,9518	$c(x \vee 1) = (1 \vee (c(x) \vee 0)) \wedge 0$	[6783:9517]
9617,9616	$(1 \vee (c(c(x)) \vee 0)) \wedge 0 = x \wedge 0$	[6391:9519]
10145	$(x \vee (y \wedge (x \vee (x \vee x)))) \wedge (c(y) \vee 1) = x$	[8822 → 393]
10355	$x \vee 1 = (y \wedge 1) \vee (c(y) \vee (x \vee 0))$	[8878 → 16]
10384	$(x \wedge 1) \vee (c(x) \vee (y \vee 0)) = y \vee 1$	[10355]
10606	$(x \wedge 1) \vee 1 = x \vee 1$	[9086 → 1763,8566,9519,9617,8142,9381]
10677,10676	$c(x \vee x) \vee 1 = c(x) \vee 1$	[663 → 10606,9229,7950]
10715,10714	$c(0 \vee (x \vee y)) \vee (x \vee 1) = c(y) \vee 1$	[8056:10677]
12016,12015	$c(x) \vee 1 = 1$	[8137 → 9200,10715]
12036,12035	$x \vee 1 = 1$	[2 → 9200,12016]
12057	$(x \vee (y \wedge (x \vee (x \vee x)))) \wedge 1 = x$	[10145:12036]
12060,12059	$(x \vee 0) \wedge 1 = x$	[9398:12036,12036,12036,12,12036,12036]
12068,12067	$x \wedge 0 = 0$	[8852:12036,46]
12074,12073	$1 \wedge (x \vee 0) = x$	[7392:12036,12036,5914,12068,6384]
12099	$(x \vee (y \wedge x)) \wedge 1 = x$	[391:12036]
12163	$(x \wedge 1) \vee (c(x) \vee (y \vee 0)) = 1$	[10384:12036]
12289	$(x \vee c(c(x \vee 0) \vee (y \vee 0))) \wedge 1 = x$	[8894:12036]
12491	$(x \vee c(y \vee (z \vee c(x)))) \wedge 1 = x$	[1212:12036,12036]
12519	$(c(x) \vee 0) \vee (y \vee (x \wedge 1)) = 1$	[138:12036]
12525	$c(x \vee x) = c(x \vee 0)$	[8261:12060]
12567	$(x \vee c(y)) \wedge ((y \wedge 1) \vee (x \vee 0)) = x$	[1878:12060]
12608	$c(x \vee 0) = c(0 \vee x)$	[9496:12068,12074]
12630	$x \wedge 1 = 1 \wedge x$	[7284:12068,12074]
12681	$c(x \vee x) \vee x = 1$	[8208:12074]
12762	$1 \wedge x = x \wedge 1$	[12630]
12928,12927	$c(x \vee (y \wedge z)) = (c(x \vee 0) \vee 0) \wedge (c(y) \vee c(z))$	[12059 → 196]
12933	$c((x \wedge 1) \vee y) = (c(x) \vee 0) \wedge (c(y \vee 0) \vee 0)$	[12059 → 196,12]
12959	$(c(x) \vee 0) \wedge (c(y \vee 0) \vee 0) = c((x \wedge 1) \vee y)$	[12933]
12968	$x \vee c(x \vee 0) = 1$	[12073 → 7744]
13051	$1 \wedge (x \vee c((0 \vee c((x \vee y) \vee (x \vee y))) \vee y)) = x$	[441 → 12630]
13192	$(x \vee (x \vee x)) \wedge 1 = x$	[8360 → 12762]
13198	$(0 \vee (x \vee (x \vee x))) \wedge 1 = x$	[4218 → 12762]
13254	$(x \vee (y \wedge 1)) \wedge (x \vee (0 \vee c(y))) = x$	[12762 → 67]
13480	$(x \vee (((x \vee c(y)) \vee (x \vee c(y))) \wedge y)) \wedge 1 = x$	[12681 → 393]
13484	$(x \vee (((y \wedge 1) \vee (y \wedge 1)) \wedge (c(y) \vee 0))) \wedge 1 = x$	[12681 → 389,12036]
13492	$(x \vee c(c((x \vee y) \vee (x \vee y)) \vee (z \vee y))) \wedge 1 = x$	[12681 → 449,12036]
13645,13644	$x \wedge (c(x) \vee 0) = 0$	[12968 → 670,5863,12060]
13708	$(x \vee (x \wedge 1)) \wedge 1 = x$	[12762 → 12099]
13752	$(c(x \vee 0) \vee 0) \wedge (c(y) \vee c(x)) = c(x \vee 0)$	[12099 → 85,12928,12,12060]
13762	$x \vee ((0 \vee c((x \vee y) \vee 0)) \vee y) = 1$	[12525 → 251]
13960	$c(0 \vee c(x)) = x \wedge 1$	[200 → 12608,6384,5477,12036]
14075,14074	$(0 \vee x) \wedge 1 = x$	[12608 → 27,28,12060]
14120,14119	$x \vee (x \vee x) = x$	[13198:14075]

14143,14142	$x \wedge 1 = x$	[13192:14120]
14147,14146	$x \vee (y \wedge x) = x$	[12057:14120,14143]
14149,14148	$1 \wedge x = x$	[8360:14120]
14153,14152	$0 \vee x = x$	[4218:14120,14149]
14183,14182	$c(c(x)) = x$	[13960:14153,14143]
14229,14228	$x \vee x = x$	[13708:14143,14143]
14256	$x \vee c(c(x \vee y) \vee (z \vee y)) = x$	[13492:14229,14143]
14259,14258	$x \vee 0 = x$	[13484:14143,14143,14229,13645,14143]
14260	$x \vee ((x \vee c(y)) \wedge y) = x$	[13480:14229,14143]
14280	$(x \vee y) \wedge (x \vee c(y)) = x$	[13254:14143,14153]
14293,14292	$c(x \vee y) = c(x) \wedge c(y)$	[12959:14259,14259,14259,14143]
14294	$(x \vee c(y)) \wedge (y \vee x) = x$	[12567:14143,14259]
14296	$c(x) \vee (y \vee x) = 1$	[12519:14259,14143]
14300	$x \vee (c(y) \wedge (c(z) \wedge x)) = x$	[12491:14293,14293,14183,14143]
14310	$x \vee (x \wedge c(y)) = x$	[12289:14259,14259,14293,14183,14143]
14320	$x \vee (c(x) \vee y) = 1$	[12163:14143,14259]
14350	$(x \vee (c(y) \wedge z)) \wedge (y \vee (x \vee c(z))) = x$	[3124:14259,14143]
14352	$(x \vee (c(y) \vee c(z))) \wedge ((y \wedge z) \vee x) = x$	[666:14143,14259]
14354	$(x \vee y) \wedge (c(y) \vee x) = x$	[425:14143,14259]
14362	$(x \vee c(y)) \vee (c(x) \wedge y) = 1$	[235:14143,14259]
14367,14366	$c(x \wedge y) = c(x) \vee c(y)$	[85:14259,14143]
14370	$x \vee ((c(x) \wedge c(y)) \vee y) = 1$	[13762:14259,14293,14153]
14374	$c(x) \wedge (c(y) \vee c(x)) = c(x)$	[13752:14259,14259,14259]
14432	$x \vee ((x \vee y) \wedge c(y)) = x$	[13051:14229,14293,14153,14293,14367,14183,14183,14149]
14465	$x \vee ((x \vee y) \wedge (c(z) \wedge c(y))) = x$	[14256:14293,14293,14367,14183,14183,14293]
14528	$x \vee (y \vee x) = y \vee x$	[14228 → 2]
14563	$x \vee (y \vee z) = z \vee (y \vee x)$	[14258 → 99,14259]
14564	$x \vee (y \vee z) = y \vee (z \vee x)$	[14258 → 16,14259]
14566	$x \vee y = y \vee x$	[14258 → 2,14259]
14600,14599	$x \vee (y \vee (z \wedge x)) = y \vee x$	[14146 → 2]
14602,14601	$(x \wedge y) \vee y = y$	[14146 → 14566]
14738	$A \vee (C \wedge (B \vee A)) \neq A \vee (B \wedge (A \vee C)) \mid \$Ans(MOD)$	[14566 → 14]
14739	$(c(x) \vee y) \wedge (y \vee x) = y$	[14566 → 7]
14743	$(c(x) \vee c(y)) \wedge (y \wedge x) = 0$	[14566 → 37]
14831	$x \vee ((y \wedge (x \vee z)) \vee z) = x \vee z$	[2 → 14601]
14859	$c(x) \wedge ((y \wedge c(x)) \vee x) = y \wedge c(x)$	[14601 → 7]
14865	$(x \vee y) \vee c(y) = 1$	[14566 → 14296]
14869	$x \vee (x \wedge y) = x$	[14182 → 14310]
14882,14881	$(x \wedge y) \vee x = x$	[14566 → 14869]
14909	$x \vee (y \vee (x \wedge z)) = y \vee x$	[14869 → 2]
14967	$(c(x) \vee y) \vee x = 1$	[14566 → 14320]
15143	$(c(x) \vee y) \vee (z \vee x) = 1$	[14967 → 2,12036]
15283	$x \vee ((c(y) \vee x) \wedge y) = x$	[14566 → 14260]
15302	$x \vee ((c(x) \wedge y) \vee c(y)) = 1$	[14260 → 14865,14367,14293,14183]
15334	$(x \vee y) \wedge (y \vee c(x)) = y$	[14566 → 14280]
15551	$c(x) \wedge (x \vee (c(x) \wedge y)) = c(x) \wedge y$	[14881 → 14294]
15747	$((x \vee y) \wedge c(y)) \vee x = x$	[14566 → 14432]
16076	$(c(x) \vee y) \vee (x \vee z) = 1$	[14566 → 15143]
16203	$x \vee (y \wedge (c(z) \wedge x)) = x$	[14182 → 14300]
16853	$x \vee (y \wedge (z \wedge x)) = x$	[14182 → 16203]
16857	$(x \wedge (y \wedge z)) \vee z = z$	[14566 → 16853]
16913,16912	$(x \wedge y) \vee (y \vee z) = y \vee z$	[14739 → 16857]
17136	$x \vee ((x \wedge c(y)) \wedge z) = x$	[16076 → 14350,14293,14183,14143]
17190	$(x \wedge y) \vee (x \vee z) = x \vee z$	[16076 → 14352,14183,14149,14183]

17370	$x \vee ((x \wedge y) \wedge z) = x$	[14182 → 17136]
17490	$c(x) \vee (((x \wedge c(y)) \vee y) \wedge x) = 1$	[15747 → 14362,14367,14293,14183,14183]
17610	$((x \wedge c(y)) \vee y) \vee x = (x \wedge c(y)) \vee y$	[14370 → 15283,14183,14149,14183]
17696	$x \wedge (c(y) \vee x) = x$	[14182 → 14374,14183,14183]
17734	$x \wedge (y \vee x) = x$	[14182 → 17696]
17786	$(x \wedge y) \wedge y = x \wedge y$	[14146 → 17734]
18418,18417	$(x \vee y) \vee z = y \vee (x \vee z)$	[14566 → 14563]
18571,18570	$(x \wedge c(y)) \vee y = y \vee x$	[17610:18418,14882]
18928	$c(x) \vee ((y \vee x) \wedge x) = 1$	[17490:18571]
18942	$c(x) \wedge (x \vee y) = y \wedge c(x)$	[14859:18571]
18945	$x \wedge c(y) = c(y) \wedge (y \vee x)$	[18942]
18989	$x \vee (y \wedge c(x)) = x \vee y$	[17786 → 18570,18571]
19021	$x \wedge (c(x) \vee y) = y \wedge x$	[18570 → 15334,14183,14147,14183]
19038,19037	$(c(x) \vee y) \wedge x = y \wedge x$	[18570 → 14294,14183,14147,14183]
19047	$x \wedge y = y \wedge (c(y) \vee x)$	[19021]
19087	$(x \vee y) \wedge y = y$	[18928 → 14354,14602,14143]
19113	$(x \vee y) \wedge x = x$	[18570 → 19087]
19122,19121	$x \wedge (y \wedge (z \wedge x)) = y \wedge (z \wedge x)$	[16853 → 19087]
19257	$c(x) \vee (y \wedge x) = c(x) \vee y$	[14182 → 18989]
19299	$x \wedge (y \vee c(x)) = y \wedge x$	[14566 → 19021]
19303,19302	$(x \vee c(y)) \wedge y = y \wedge (x \vee c(y))$	[14528 → 19021]
19324	$x \wedge y = y \wedge (x \vee c(y))$	[19299]
19357	$x \vee (y \wedge (x \vee c(y))) = x$	[14260:19303]
19592	$(c(x) \wedge (x \vee y)) \vee x = x \vee y$	[19047 → 18570,14183]
19636	$(x \wedge (c(x) \vee y)) \vee y = y$	[19047 → 14881]
19852	$(x \wedge y) \vee (y \wedge (c(y) \vee x)) = x \wedge y$	[18570 → 19357,14183,14183]
20391,20390	$(x \vee (y \wedge z)) \wedge (x \vee z) = x \vee (y \wedge z)$	[14599 → 17734]
20402	$(x \vee (y \wedge c(z))) \wedge z = z \wedge (x \vee c(z))$	[14599 → 19037,19303]
20556	$(x \vee (c(y) \wedge z)) \wedge y = y \wedge (x \vee c(y))$	[14909 → 19037,19303]
20672	$x \vee ((x \wedge y) \vee c(y)) = (x \wedge y) \vee c(y)$	[15302 → 19636,14143,14183,14183]
20674	$((x \wedge y) \vee c(y)) \wedge x = x$	[15302 → 19037,14149,14183]
20678	$((x \wedge (y \vee c(x))) \vee c(x)) \wedge y = y$	[19324 → 20674]
20707,20706	$x \vee (c(x) \wedge y) = x \vee y$	[14280 → 20674,14293,14183,20391]
20713,20712	$c(x) \wedge (x \vee y) = c(x) \wedge y$	[15551:20707]
20718	$(c(x) \wedge y) \vee x = x \vee y$	[19592:20713]
20720	$x \wedge c(y) = c(y) \wedge x$	[18945:20713]
20740	$x \wedge y = y \wedge x$	[14182 → 20720,14183]
20742,20741	$c(x) \wedge (y \vee x) = c(x) \wedge y$	[19324 → 20720,14183]
20768,20767	$x \wedge (y \vee c(x)) = x \wedge y$	[19324 → 20740]
20770,20769	$x \wedge (c(x) \vee y) = x \wedge y$	[19047 → 20740]
20775	$((x \wedge y) \vee c(x)) \wedge y = y$	[20678:20768]
20778,20777	$(x \vee (c(y) \wedge z)) \wedge y = y \wedge x$	[20556:20768]
20780,20779	$(x \vee (y \wedge c(z))) \wedge z = z \wedge x$	[20402:20768]
20783	$(x \wedge y) \vee (y \wedge x) = x \wedge y$	[19852:20770]
20960,20959	$(x \vee c(y)) \wedge y = y \wedge x$	[20740 → 20767]
21072	$(x \wedge y) \wedge (y \wedge x) = y \wedge x$	[14743 → 20775,14293,14183,14183,14153]
21087	$x \vee ((y \wedge x) \vee c(y)) = (y \wedge x) \vee c(y)$	[20775 → 14881]
21465,21464	$x \vee ((y \wedge x) \vee z) = x \vee z$	[2 → 16912]
21469,21468	$(x \wedge y) \vee c(x) = y \vee c(x)$	[21087:21465]
21588,21587	$x \vee ((x \wedge y) \vee z) = x \vee z$	[2 → 17190]
21594,21593	$(x \wedge y) \vee c(y) = x \vee c(y)$	[20672:21588]
22207	$(x \wedge c(y)) \vee (x \wedge y) = x$	[20769 → 19257,14293,14183,14293,14183,14882]
22209	$(c(x) \wedge y) \vee (y \wedge x) = y$	[20767 → 19257,14293,14183,14293,14183,14602]
22240	$(x \wedge c(y)) \vee (y \wedge x) = x$	[20740 → 22207]

22267	$(x \wedge y) \vee (y \wedge c(x)) = y$	[14182 → 22209]
22271	$(x \wedge y) \vee (c(y) \wedge x) = x$	[14566 → 22209]
22394	$(x \wedge y) \vee (c(x) \wedge y) = y$	[20740 → 22267]
22462	$x \vee ((x \vee y) \wedge (z \wedge c(y))) = x$	[14182 → 14465]
23045,23044	$(x \wedge y) \vee ((y \wedge x) \vee z) = (y \wedge x) \vee z$	[20783 → 18417]
23048	$x \vee (y \wedge z) = (z \wedge y) \vee x$	[20783 → 14564,23045]
23056	$(x \wedge y) \vee z = z \vee (y \wedge x)$	[23048]
23273	$(x \wedge y) \vee ((y \wedge x) \wedge z) = x \wedge y$	[21072 → 17370]
23304,23303	$(x \vee (y \vee c(z))) \wedge (z \vee x) = (y \wedge z) \vee x$	[21464 → 14294,18418,21594]
23439,23438	$((c(x) \wedge y) \vee z) \wedge x = z \wedge x$	[21587 → 19037,19038]
24842	$x \wedge (y \wedge (x \vee c(z))) = y \wedge x$	[18570 → 20777,23439,14367,14183]
25351	$x \vee ((y \vee x) \wedge (z \wedge c(y))) = x$	[14566 → 22462]
26810,26809	$x \wedge (y \wedge (x \vee z)) = y \wedge x$	[14182 → 24842]
26841	$(x \wedge y) \wedge (z \wedge x) = z \wedge (x \wedge y)$	[22271 → 26809]
26852	$x \wedge (y \wedge (z \vee x)) = y \wedge x$	[14909 → 26809]
26870	$x \wedge ((x \vee y) \wedge z) = x \wedge z$	[20959 → 26809,14293,20778]
26873	$(x \wedge (y \vee z)) \wedge y = x \wedge y$	[17786 → 26809,26810]
26984,26983	$(x \wedge (y \vee z)) \vee c(y) = x \vee c(y)$	[26809 → 21468,21594]
27027	$x \wedge ((y \vee x) \wedge z) = x \wedge z$	[20959 → 26852,14293,20780]
27097	$x \vee (y \wedge (z \vee c(x))) = x \vee y$	[26852 → 20718,18571]
27141	$(x \wedge (y \vee z)) \vee c(z) = x \vee c(z)$	[26852 → 21468,21594]
27179,27178	$(x \wedge y) \wedge (y \wedge z) = (x \wedge y) \wedge z$	[22394 → 26870]
27267	$(((x \vee y) \wedge z) \wedge c(x)) \vee (x \wedge z) = (x \vee y) \wedge z$	[26870 → 22240]
27343,27342	$(x \wedge y) \wedge z = x \wedge (y \wedge z)$	[22271 → 26873,27179]
27374	$((x \vee y) \wedge (z \wedge c(x))) \vee (x \wedge z) = (x \vee y) \wedge z$	[27267:27343]
27453	$x \wedge (y \wedge z) = y \wedge (z \wedge x)$	[26841:27343,19122]
27777	$(x \wedge y) \vee (y \wedge (x \wedge z)) = x \wedge y$	[23273:27343]
28023,28022	$((x \vee y) \wedge z) \vee c(y) = z \vee c(y)$	[27027 → 21468,21469]
28504,28503	$(x \vee y) \wedge (z \wedge c(x)) = y \wedge (z \wedge c(x))$	[25351 → 14294,28023,20960]
28506	$(x \wedge (y \wedge c(z))) \vee (z \wedge y) = (z \vee x) \wedge y$	[27374:28504]
28508	$(x \vee y) \wedge z = (y \wedge (z \wedge c(x))) \vee (x \wedge z)$	[28506]
28889	$((x \vee c(y)) \wedge z) \vee y = y \vee z$	[23048 → 27097]
28891	$(x \wedge (y \vee c(z))) \vee z = z \vee x$	[14566 → 27097]
30251,30250	$(x \wedge (y \wedge (z \vee c(u)))) \vee u = u \vee (x \wedge y)$	[27453 → 28889]
30648	$(x \wedge (y \vee z)) \vee z = (x \wedge y) \vee z$	[14831 → 14294,18418,26984,23304]
32819	$(x \vee c(y)) \wedge (y \vee (x \wedge (z \vee y))) = x \wedge (z \vee y)$	[27141 → 14354,14183]
33046	$(x \wedge ((x \wedge y) \vee z)) \vee (x \wedge y) = x \wedge ((x \wedge y) \vee z)$	[19113 → 27777]
33567,33566	$(x \wedge (y \vee z)) \vee y = y \vee (x \wedge z)$	[28891 → 30648,30251]
33581,33580	$(x \wedge (y \wedge c(z))) \vee (z \wedge y) = (x \wedge y) \vee (z \wedge y)$	[22240 → 30648]
33618,33617	$x \vee (y \wedge (z \vee x)) = (y \wedge z) \vee x$	[14566 → 30648]
33632,33631	$x \wedge ((x \wedge y) \vee z) = (x \wedge y) \vee (x \wedge z)$	[33046:33567]
33641,33640	$(x \vee y) \wedge z = (y \wedge z) \vee (x \wedge z)$	[28508:33581]
33668,33667	$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$	[32819:33618,33641,20742,33632,16913]
33669	$(C \wedge B) \vee A \neq A \vee (B \wedge C) \mid \$Ans(MOD)$	[14738:33668,14600,33668,21465]
33670	$\$Ans(MOD)$	[33669,23056]

Proof BA3

1	$A \vee (A \wedge B) \neq A \mid \$Ans(B1)$	[]
2	$c(c(A)) \neq A \mid \$Ans(CC)$	[]
4	$x \vee (y \vee z) = y \vee (x \vee z)$	[]
5	$x \wedge y = c(c(x) \vee c(y))$	[]

6	$c(c(x) \vee c(y)) = x \wedge y$	[5]
9	$(x \vee c(y)) \wedge (x \vee y) = x$	[]
12,11	$x \vee c(x) = 1$	[]
14,13	$c(1) = 0$	[]
15	$x \vee (y \vee (z \vee u)) = z \vee (x \vee (y \vee u))$	[4 → 4]
16	$x \vee (y \vee (z \vee u)) = y \vee (z \vee (x \vee u))$	[15]
18,17	$1 \vee 0 = 1$	[13 → 11]
19	$x \vee (y \vee c(x)) = y \vee 1$	[11 → 4]
21	$x \vee 1 = 1 \vee (x \vee 0)$	[17 → 4]
22	$1 \vee (x \vee 0) = x \vee 1$	[21]
23	$c(0 \vee c(x)) = 1 \wedge x$	[13 → 6]
25	$c((x \wedge y) \vee c(z)) = (c(x) \vee c(y)) \wedge z$	[6 → 6]
28,27	$c(c(x) \vee 0) = x \wedge 1$	[13 → 6]
29	$c(c(x) \vee (y \wedge z)) = x \wedge (c(y) \vee c(z))$	[6 → 6]
31	$x \wedge c(x) = 0$	[11 → 6,14]
34,33	$(c(x) \vee c(y)) \vee (x \wedge y) = 1$	[6 → 11]
35	$1 \wedge 0 = 0$	[13 → 31]
39	$x \vee (1 \vee (y \vee 0)) = y \vee (x \vee 1)$	[21 → 4]
40	$x \vee (y \vee 1) = y \vee (1 \vee (x \vee 0))$	[39]
42,41	$(x \vee 0) \wedge (x \vee 1) = x$	[13 → 9]
43	$(x \vee (y \wedge z)) \wedge (x \vee (c(y) \vee c(z))) = x$	[6 → 9]
46,45	$1 \wedge (x \vee x) = x$	[11 → 9]
47	$(x \vee 0) \wedge (1 \vee (x \vee 0)) = x$	[21 → 9,14]
49	$(1 \vee c(0)) \wedge 1 = 1$	[17 → 9]
52,51	$(x \vee c(c(x))) \wedge 1 = x$	[11 → 9]
53	$(x \vee c(y \vee z)) \wedge (y \vee (x \vee z)) = x$	[4 → 9]
56,55	$1 \wedge (x \vee ((x \vee y) \vee y)) = x \vee y$	[4 → 45]
59	$x \vee (y \vee 1) = 1 \vee (x \vee (y \vee 0))$	[22 → 4]
60	$1 \vee (x \vee (y \vee 0)) = x \vee (y \vee 1)$	[59]
62	$x \vee (y \vee (1 \vee (z \vee 0))) = z \vee (x \vee (y \vee 1))$	[21 → 15]
64	$x \vee (y \vee (z \vee c(x))) = y \vee (z \vee 1)$	[11 → 15]
68	$x \vee (y \vee (z \vee u)) = z \vee (y \vee (x \vee u))$	[4 → 15]
70	$x \vee (y \vee (z \vee 1)) = y \vee (z \vee (1 \vee (x \vee 0)))$	[62]
74	$1 \wedge (x \vee ((y \vee (x \vee z)) \vee (y \vee z))) = y \vee (x \vee z)$	[15 → 45]
76	$(x \vee c(y \vee (z \vee u))) \wedge (z \vee (x \vee (y \vee u))) = x$	[15 → 9]
83,82	$c(0 \vee 0) = 1 \wedge 1$	[13 → 23]
84	$c(0 \vee (x \wedge y)) = 1 \wedge (c(x) \vee c(y))$	[6 → 23]
86	$(x \vee (1 \wedge y)) \wedge (x \vee (0 \vee c(y))) = x$	[23 → 9]
90	$(0 \vee c(x)) \vee (1 \wedge x) = 1$	[23 → 11]
92	$(x \vee (1 \wedge 1)) \wedge (x \vee (0 \vee 0)) = x$	[82 → 9]
96	$(0 \vee 0) \vee (1 \wedge 1) = 1$	[82 → 11]
105,104	$c((x \wedge y) \vee 0) = (c(x) \vee c(y)) \wedge 1$	[6 → 27]
108	$(c(x) \vee 0) \wedge (x \wedge 1) = 0$	[27 → 31]
110	$(c(x) \vee 0) \vee (x \wedge 1) = 1$	[27 → 11]
116	$x \vee (y \vee 1) = 1 \vee (y \vee (x \vee 0))$	[22 → 16]
124	$1 \vee (x \vee (y \vee 0)) = y \vee (x \vee 1)$	[116]
126	$1 \wedge (x \vee (y \vee ((x \vee (y \vee z)) \vee z))) = x \vee (y \vee z)$	[16 → 45]
130	$(x \vee c(y \vee (z \vee u))) \wedge (y \vee (z \vee (x \vee u))) = x$	[16 → 9]
133	$((0 \vee 0) \vee c(1 \wedge 1)) \wedge 1 = 0 \vee 0$	[82 → 51]
167	$c(x \vee c(y)) = (0 \vee c(x \vee x)) \wedge y$	[45 → 25,14]
169,168	$c(x \vee c(y)) = (c(x \vee 0) \vee c(x \vee 1)) \wedge y$	[41 → 25]
171,170	$((1 \wedge 1) \vee c(0 \vee 1)) \wedge x = 1 \wedge x$	[35 → 25,169,83,14,12]
181,180	$c((x \wedge y) \vee (z \wedge u)) = (c(x) \vee c(y)) \wedge (c(z) \vee c(u))$	[6 → 25]
184	$(0 \vee c(x \vee x)) \wedge y = (c(x \vee 0) \vee c(x \vee 1)) \wedge y$	[167,169]

190	$((x \wedge 1) \vee c(c(x) \vee 1)) \wedge y = x \wedge y$	[6:169,28]
192	$(0 \vee (1 \wedge 1)) \vee 0 = 1$	[82 → 90,46]
196	$(0 \vee c(x \vee x)) \vee x = 1$	[45 → 90]
200	$1 \wedge ((0 \vee (1 \wedge 1)) \vee 1) = 0 \vee (1 \wedge 1)$	[192 → 41]
203,202	$(0 \vee (1 \wedge 1)) \vee 1 = 1 \vee 1$	[192 → 22]
209	$0 \vee (1 \wedge 1) = 1$	[200:203,46]
214	$(0 \vee c(1 \wedge 1)) \wedge 1 = 0$	[209 → 9]
216	$0 \vee (x \vee (1 \wedge 1)) = x \vee 1$	[209 → 4]
218	$c(c(x) \vee y) = x \wedge (((c(y \vee 0) \vee c(y \vee 1)) \wedge c(y)) \vee 0)$	[51 → 29,169,14]
220,219	$c(c(x) \vee 1) = x \wedge (((0 \vee c(1 \vee 1)) \wedge 0) \vee 0)$	[49 → 29,169,18,14,14]
221	$c(c(x) \vee y) = x \wedge (0 \vee c(y \vee y))$	[45 → 29,14]
222	$c(c(x) \vee y) = x \wedge (c(y \vee 0) \vee c(y \vee 1))$	[41 → 29]
224	$x \wedge (((c(y \vee 0) \vee c(y \vee 1)) \wedge c(y)) \vee 0) = c(c(x) \vee y)$	[218]
226,225	$((x \wedge 1) \vee (x \wedge (((0 \vee c(1 \vee 1)) \wedge 0) \vee 0))) \wedge y = x \wedge y$	[190:220]
227	$x \wedge (0 \vee c(y \vee y)) = c(c(x) \vee y)$	[221]
242	$(c(x) \vee (y \wedge z)) \vee (x \wedge (c(y) \vee c(z))) = 1$	[29 → 11]
248	$((x \wedge 1) \vee 0) \wedge ((c(x) \vee 0) \wedge 1) = 0$	[27 → 108]
258	$(((c(x \vee 0) \vee c(x \vee 1)) \wedge c(x)) \vee 0) \vee x = 1$	[51 → 110,169]
282	$(c(x \vee 0) \vee c(x \vee 1)) \vee x = 1$	[41 → 33]
286	$(c(x) \vee c(y)) \vee (z \vee (u \vee (x \wedge y))) = z \vee (u \vee 1)$	[33 → 15]
290	$(0 \vee c(x \vee ((y \vee (x \vee z)) \vee (y \vee z)))) \vee (y \vee (x \vee z)) = 1$	[15 → 196]
305,304	$(0 \vee (x \wedge x)) \vee 1 = x \vee 1$	[196 → 19,169,28,220,226]
307	$x \vee (y \vee ((0 \vee c((x \vee z) \vee (x \vee z))) \vee z)) = y \vee 1$	[196 → 15]
315	$x \vee 1 = (0 \vee c(y \vee y)) \vee (x \vee y)$	[196 → 4]
319	$(0 \vee c(x \vee x)) \vee (y \vee x) = y \vee 1$	[315]
321,320	$((1 \wedge (1 \wedge 1)) \vee 0) \vee 0 = 1$	[214 → 110,169,83,171]
367,366	$(0 \vee 0) \vee 1 = 0 \vee 1$	[196 → 216,181,14,14,14,14,305]
386	$x \vee (y \vee 1) = x \vee (1 \vee (y \vee 0))$	[4 → 40]
420,419	$(1 \wedge (1 \wedge 1)) \vee 0 = 1$	[320 → 47,321,46]
422	$1 \wedge (1 \wedge 1) = 1$	[419 → 47,420,46]
430	$(x \vee y) \wedge (x \vee (0 \vee c(y \vee y))) = x$	[45 → 43,14]
438	$1 \wedge (0 \vee (0 \vee 0)) = 0$	[209 → 43,14,14]
452	$(x \vee (y \wedge x)) \wedge (c(y) \vee 1) = x$	[19 → 43]
454	$(x \vee (y \wedge z)) \wedge (c(y) \vee (x \vee c(z))) = x$	[4 → 43]
464	$(x \vee 1) \wedge (x \vee (0 \vee c(1 \wedge 1))) = x$	[422 → 43,14]
466	$x \wedge (0 \vee c(1 \wedge 1)) = x \wedge (((0 \vee c(1 \vee 1)) \wedge 0) \vee 0)$	[422 → 29,220,14]
469,468	$(0 \vee c(0 \vee (0 \vee 0))) \vee 0 = 1$	[438 → 90]
503	$(x \vee (1 \wedge 1)) \wedge (0 \vee (x \vee 0)) = x$	[82 → 53]
639	$((1 \wedge 1) \vee 1) \wedge (0 \vee 1) = 1 \wedge 1$	[422 → 452,14]
651	$((x \vee x) \vee x) \wedge (0 \vee 1) = x \vee x$	[45 → 452,14]
714	$0 \vee c(0 \vee (0 \vee 0)) = 1$	[468 → 47,469,46]
744,743	$(0 \vee c((x \vee 0) \vee (x \vee 0))) \vee (x \vee 1) = 1 \vee 1$	[196 → 60]
903,902	$((1 \wedge 1) \vee c(0 \vee 1)) \vee 0 = 1$	[82 → 282]
946	$(1 \wedge 1) \vee c(0 \vee 1) = 1$	[282 → 47,83,903,46,83]
1015	$(1 \wedge 1) \vee c(1 \vee (0 \vee 0)) = 1$	[21 → 946]
1017	$1 \wedge (0 \vee ((1 \wedge 1) \vee 1)) = 1 \wedge 1$	[946 → 53]
1020,1019	$(1 \wedge 1) \vee 1 = (0 \vee 1) \vee 1$	[946 → 19]
1037,1036	$1 \wedge 1 = 0 \vee 1$	[1017:1020,56]
1043,1042	$((0 \vee 1) \vee 1) \wedge (0 \vee 1) = 0 \vee 1$	[639:1037,1037]
1044	$(0 \vee 1) \vee c(1 \vee (0 \vee 0)) = 1$	[1015:1037]
1092	$(x \vee (0 \vee 1)) \wedge (0 \vee (x \vee 0)) = x$	[503:1037]
1105,1104	$x \wedge (((0 \vee c(1 \vee 1)) \wedge 0) \vee 0) = x \wedge (0 \vee c(0 \vee 1))$	[466:1037]
1106	$(x \vee 1) \wedge (x \vee (0 \vee c(0 \vee 1))) = x$	[464:1037]
1139,1138	$0 \vee (0 \vee 1) = 1$	[209:1037]

1144	$((0 \vee 0) \vee c(0 \vee 1)) \wedge 1 = 0 \vee 0$	[133:1037]
1146	$(0 \vee 0) \vee (0 \vee 1) = 1$	[96:1037]
1148	$(x \vee (0 \vee 1)) \wedge (x \vee (0 \vee 0)) = x$	[92:1037]
1151,1150	$c(0 \vee 0) = 0 \vee 1$	[82:1037]
1161,1160	$((x \wedge 1) \vee (x \wedge (0 \vee c(0 \vee 1)))) \wedge y = x \wedge y$	[225:1105]
1163,1162	$c(c(x) \vee 1) = x \wedge (0 \vee c(0 \vee 1))$	[219:1105]
1164	$(1 \vee (0 \vee 1)) \wedge (0 \vee 1) = 1$	[1036 → 452,14]
1166	$c(c(x) \vee (0 \vee 1)) = x \wedge (0 \vee 0)$	[1036 → 29,14,14]
1181	$0 \vee (1 \vee (0 \vee 0)) = 1$	[21 → 1138]
1190,1189	$0 \vee (0 \vee (x \vee 1)) = x \vee 1$	[1138 → 16]
1203	$(0 \vee 0) \vee (0 \vee (x \vee 1)) = x \vee 1$	[1146 → 16]
1286	$0 \vee (1 \vee (x \vee (0 \vee 0))) = x \vee 1$	[1181 → 16]
1295	$0 \vee (x \vee (1 \vee (0 \vee 0))) = x \vee 1$	[1181 → 4]
1459,1458	$(0 \vee (1 \vee 1)) \wedge (0 \vee 1) = 1$	[40 → 1164,18]
1550,1549	$x \wedge (c(0 \vee (1 \vee 1)) \vee c(0 \vee 1)) = x \wedge (0 \vee c(0 \vee 1))$	[1458 → 29,1163]
1560,1559	$(1 \vee (0 \vee 0)) \vee 1 = (0 \vee 1) \vee 1$	[1044 → 19]
1896	$(x \vee (1 \vee (((x \vee 1) \vee (x \vee 1)) \vee 0))) \wedge (0 \vee 1) = (x \vee 1) \vee (x \vee 1)$	[40 → 651]
2005	$(c((0 \vee 1) \vee 1) \vee c(0 \vee 1)) \vee (0 \vee 1) = 1$	[1042 → 33]
2135	$(0 \vee (x \vee 1)) \wedge (0 \vee (x \vee 0)) = x$	[4 → 1092]
2213	$(0 \vee (x \vee 1)) \wedge (x \vee (0 \vee 0)) = x$	[4 → 1148]
2216,2215	$0 \vee c((0 \vee 0) \vee (0 \vee 0)) = (1 \vee 1) \wedge 1$	[196 → 1148,744]
2235	$c(0 \vee (c(x) \vee 1)) = x \wedge (0 \vee 0)$	[4 → 1166]
2361,2360	$(c((0 \vee 0) \vee 0) \vee c(0 \vee 1)) \vee 1 = 0 \vee (1 \vee 1)$	[282 → 1286,367]
2384,2383	$1 \wedge (0 \vee c(0 \vee 1)) = c(0 \vee 1)$	[1458 → 84,1550]
2405	$c(0 \vee x) = 1 \wedge (0 \vee c(x \vee x))$	[45 → 84,14]
2411	$1 \wedge (0 \vee c(x \vee x)) = c(0 \vee x)$	[2405]
2535	$(x \vee (1 \wedge y)) \wedge (0 \vee (x \vee c(y))) = x$	[4 → 86]
2733	$1 \wedge ((1 \vee (0 \vee 0)) \vee (((0 \vee 1) \vee 1) \vee 1)) = (0 \vee 1) \vee 1$	[1559 → 55,1560]
2816,2815	$c((0 \vee 0) \vee 0) \vee c(0 \vee 1) = (1 \vee 1) \wedge 1$	[282 → 2213,367,2361,1190,367]
3021	$c(1 \vee (0 \vee 0)) = 1 \wedge (0 \vee c(1 \vee 1))$	[21 → 2405]
3107	$0 \vee (1 \wedge ((1 \vee 1) \wedge 1)) = 1$	[2405 → 714,2216]
3185	$1 \wedge (0 \vee c((1 \vee 1) \wedge 1)) = 0$	[3107 → 84,14,14]
3208	$0 \vee (x \vee (1 \wedge ((1 \vee 1) \wedge 1))) = x \vee 1$	[3107 → 4]
3210	$((0 \vee c((1 \vee 1) \wedge 1)) \vee 0) \wedge (0 \vee 1) = 0 \vee c((1 \vee 1) \wedge 1)$	[3185 → 452,14]
3865	$x \wedge (0 \vee c(y \vee ((1 \vee (y \vee 0)) \vee 1))) = c(c(x) \vee (1 \vee (y \vee 0)))$	[39 → 227]
3867,3866	$x \wedge (0 \vee c(1 \vee 1)) = x \wedge (0 \vee c(0 \vee 1))$	[21 → 227,18,1163]
3877	$c(c(x) \vee (1 \vee (y \vee 0))) = x \wedge (0 \vee c(y \vee ((1 \vee (y \vee 0)) \vee 1)))$	[3865]
3881,3880	$c(1 \vee (0 \vee 0)) = c(0 \vee 1)$	[3021:3867,2384]
4923,4922	$(0 \vee (x \wedge x)) \vee (y \vee 1) = x \vee (y \vee 1)$	[319 → 64,169,28,1163,1161]
5028	$(x \vee (1 \vee (y \vee 0))) \wedge (x \vee (0 \vee c(y \vee ((1 \vee (y \vee 0)) \vee 1)))) = x$	[39 → 430]
5587	$0 \vee ((0 \vee 1) \wedge (0 \vee 1)) = 0 \vee 1$	[319 → 1106,169,28,1163,1161,305,1043,169,28,1163,1161]
5612,5611	$(c(0 \vee 1) \vee c(0 \vee 1)) \vee (0 \vee 1) = 0 \vee 1$	[5587 → 319,181,4923]
5840	$((0 \vee 0) \vee 0) \wedge (((((1 \vee 1) \wedge 1) \wedge (0 \vee 1)) \vee 0) \wedge 1) = 0$	[1144 → 248,169,367,2816]
5844	$c((0 \vee 0) \vee 0) = ((((1 \vee 1) \wedge 1) \wedge (0 \vee 1)) \vee 0) \wedge 1$	[1144 → 104,169,367,2816,14]
5866,5865	$c((0 \vee 1) \vee 1) = (0 \vee 0) \wedge (0 \vee c(0 \vee 1))$	[1150 → 1162]
5883	$(((0 \vee 0) \wedge (0 \vee c(0 \vee 1))) \vee c(0 \vee 1)) \vee (0 \vee 1) = 1$	[2005:5866]
6410	$c(x \vee (0 \vee 1)) = (c(x \vee 0) \vee c(x \vee 1)) \wedge (0 \vee 0)$	[1150 → 168]
6422	$x \wedge (0 \vee (y \wedge y)) = x \wedge y$	[221 → 168,169,28,1163,1161,28,1163,1161]
7247	$x \wedge ((y \wedge c(c(y))) \vee 0) = x \wedge y$	[168 → 218,28,1163,1161,28,1163,1161]
7474	$c((x \wedge (0 \vee 0)) \vee y) = (0 \vee (c(x) \vee 1)) \wedge (c(y \vee 0) \vee c(y \vee 1))$	[2235 → 222]
9519	$((x \wedge c(y)) \vee (x \wedge y)) \vee c(x) = 1$	[9 → 242,169,28,1163,1161]
9633	$((1 \wedge x) \vee c(0 \vee x)) \vee 0 = 1$	[2411 → 9519,169,1151,12,46,14]
10131,10130	$((1 \wedge x) \vee c(0 \vee x)) \vee 1 = 1 \vee 1$	[9633 → 319,1151,1139]
10150	$(1 \wedge x) \vee c(0 \vee x) = 1$	[9633 → 2135,10131,1459]

10186	$(1 \wedge x) \vee c(0 \vee ((x \wedge c(c(x))) \vee 0)) = 1$	[7247 → 10150]
10212	$c(0 \vee x) \vee c(0 \vee (((c(x \vee 0) \vee c(x \vee 1)) \wedge c(x)) \vee 0)) = 1$	[224 → 10150,14]
10220	$x \vee c(0 \vee (x \vee x)) = 1$	[45 → 10150]
10301,10300	$(1 \wedge x) \vee 1 = (0 \vee x) \vee 1$	[10150 → 319,169,28,1163,1161,305]
10440	$(0 \vee (x \vee x)) \vee 1 = x \vee 1$	[10220 → 319,169,28,1163,1161,305]
10444,10443	$1 \wedge (0 \vee (x \vee (x \vee x))) = x$	[10220 → 130]
10445	$1 \wedge (x \vee (x \vee (0 \vee x))) = x$	[10220 → 76]
10688	$(0 \vee ((x \wedge c(c(x))) \vee 0)) \vee 1 = (0 \vee x) \vee 1$	[7247 → 10300,10301]
10690	$(0 \vee (0 \vee (x \wedge x))) \vee 1 = (0 \vee x) \vee 1$	[6422 → 10300,10301]
10848,10847	$((0 \vee 1) \vee 1) \vee 1 = (0 \vee 1) \vee 1$	[1295 → 10440,1560,1560]
10902,10901	$1 \wedge ((1 \vee (0 \vee 0)) \vee ((0 \vee 1) \vee 1)) = (0 \vee 1) \vee 1$	[2733:10848]
10903	$0 \vee (1 \vee 1) = 1$	[10440 → 126,10444]
11101,11100	$((x \wedge c(c(x))) \vee 0) \vee 1 = x \vee 1$	[258 → 19,28,1163,1161]
11131,11130	$1 \vee 1 = 0 \vee 1$	[10903 → 1203,367]
11458	$c((0 \vee 0) \vee 0) = ((((0 \vee 1) \wedge 1) \wedge (0 \vee 1)) \vee 0) \wedge 1$	[5844:11131]
11460	$((0 \vee 0) \vee 0) \wedge (((((0 \vee 1) \wedge 1) \wedge (0 \vee 1)) \vee 0) \wedge 1) = 0$	[5840:11131]
11596	$((0 \vee c((0 \vee 1) \wedge 1)) \vee 0) \wedge (0 \vee 1) = 0 \vee c((0 \vee 1) \wedge 1)$	[3210:11131,11131]
11598	$0 \vee (x \vee (1 \wedge ((0 \vee 1) \wedge 1))) = x \vee 1$	[3208:11131]
11992,11991	$(0 \vee 1) \vee 1 = 1 \vee (0 \vee 0)$	[1181 → 10445,1560,10902]
12052,12051	$(0 \vee 0) \wedge (0 \vee c(0 \vee 1)) = c(0 \vee 1)$	[5865:11992,3881]
12140,12139	$0 \vee 1 = 1$	[5883:12052,5612]
12253,12252	$0 \vee (x \vee 1) = x \vee 1$	[11598:12140,1037,12140,1037,12140]
12255,12254	$((0 \vee 0) \vee 0) \wedge 1 = 0 \vee 0$	[11596:12140,1037,12140,14,12140,12140,1037,12140,14]
12354,12353	$0 \vee 0 = 0$	[11460:12140,1037,12140,12140,1037,12140,18,1037,12140,12255]
12356,12355	$c(0) = 1$	[11458:12354,12354,12140,1037,12140,12140,1037,12140,18,1037,12140]
12406,12405	$1 \vee 1 = 1$	[11130:12140]
12584,12583	$(c(x \vee 0) \vee c(x \vee 1)) \wedge 0 = c(x \vee 1)$	[6410:12140,12354]
12810	$(x \vee (1 \vee (((x \vee 1) \vee (x \vee 1)) \vee 0))) \wedge 1 = (x \vee 1) \vee (x \vee 1)$	[1896:12140]
12827,12826	$c(c(x) \vee 1) = x \wedge 0$	[1162:12140,14,12354]
12829,12828	$((x \wedge 1) \vee (x \wedge 0)) \wedge y = x \wedge y$	[1160:12140,14,12354]
12896	$c((x \wedge 0) \vee y) = (c(x) \vee 1) \wedge (c(y \vee 0) \vee c(y \vee 1))$	[7474:12354,12253]
13249,13248	$0 \vee (x \vee (y \vee 0)) = x \vee (y \vee 0)$	[12353 → 15]
13257,13256	$0 \vee (x \vee 0) = x \vee 0$	[12353 → 4]
13315,13314	$(0 \vee x) \vee 1 = x \vee 1$	[10688:13257,11101]
13316	$c(0 \vee x) \vee x = 1$	[10212:13257,105,169,28,12827,12829,42,52]
13318	$(1 \wedge x) \vee c(x) = 1$	[10186:13257,105,52]
13448	$(x \wedge x) \vee 1 = x \vee 1$	[10690:13315,13315,13315]
13452	$(x \vee x) \vee 1 = x \vee 1$	[10440:13315]
13859	$x \vee (c(x \vee 0) \vee 1) = 1$	[13316 → 124,12406,13257]
13861	$c(x \vee 0) \vee (x \vee 1) = 1$	[13316 → 60,12406,13257]
14031	$x \vee c(x \vee x) = 1$	[45 → 13318]
14186,14185	$(x \vee x) \wedge 1 = x$	[14031 → 2535,46,12140]
14305,14304	$(x \vee ((x \vee y) \vee y)) \wedge 1 = x \vee y$	[4 → 14185]
14332	$(c(x \vee x) \vee 0) \vee x = 1$	[14185 → 110]
14719	$c(x \vee 0) \vee (x \vee 0) = 1$	[13256 → 13316]
14843	$(1 \vee (x \vee 0)) \vee 1 = (x \vee 1) \vee 1$	[386 → 13314,13249]
14882,14881	$1 \vee ((0 \vee x) \vee 0) = x \vee 1$	[21 → 13314]
15156,15155	$1 \vee ((x \vee x) \vee 0) = x \vee 1$	[21 → 13452]
15158,15157	$(x \vee 1) \vee (x \vee 1) = x \vee 1$	[12810:15156,14305]
15355,15354	$1 \wedge (x \vee 1) = x \vee 1$	[286 → 74,34,15158,34]
15822	$x \vee (y \vee (c(y \vee ((x \vee (y \vee 1)) \vee (x \vee 1))) \vee 1)) = 1$	[70 → 290,14882]
15828	$x \vee (c(x \vee ((1 \vee (x \vee 0)) \vee 1)) \vee 1) = 1$	[39 → 290,18,13315]
15842	$x \vee (y \vee (c(x \vee ((1 \vee (x \vee 0)) \vee 1)) \vee 1)) = y \vee 1$	[290 → 62,18,13315]
16165	$(x \vee c(y \vee (z \vee (c(x \vee 0) \vee 1)))) \wedge (y \vee (z \vee 1)) = x$	[13859 → 130]

16209	$c(x) \vee (x \vee 1) = 1$	[13448 → 13861,105,14186]
16220,16219	$c(x \vee 1) \vee 1 = c(x \vee 0) \vee 1$	[13861 → 319,15158,13315]
16363	$x \vee (c(x) \vee 1) = 1$	[4 → 16209]
16366,16365	$c(x \vee 0) \vee 1 = c(x) \vee 1$	[16209 → 319,15158,13315,16220]
16466,16465	$c(x \vee 1) \vee 1 = c(x) \vee 1$	[16219:16366]
16469	$(x \vee c(y \vee (z \vee (c(x) \vee 1)))) \wedge (y \vee (z \vee 1)) = x$	[16165:16366]
16500	$(x \wedge 0) \vee 1 = x \vee 1$	[16363 → 319,15158,12827,13315]
16723,16722	$c(x \vee x) \vee 1 = c(x) \vee 1$	[184 → 16500,12584,16466,13315]
17210	$(c(x \vee ((1 \vee (x \vee 0)) \vee 1)) \vee 0) \vee (1 \vee (x \vee 0)) = 1$	[39 → 14332]
17536	$x \vee 1 = y \vee (c(y \vee 0) \vee (x \vee 0))$	[14719 → 68]
17573	$x \vee (c(x \vee 0) \vee (y \vee 0)) = y \vee 1$	[17536]
17794,17793	$(x \vee 1) \vee 1 = x \vee 1$	[16209 → 307,15158,169,12354,12356,12140,14,18,15355]
17796,17795	$x \vee (c(x \vee y) \vee 1) = c(y) \vee 1$	[319 → 307,13315,16723,13315,16723]
17875,17874	$(1 \vee (x \vee 0)) \vee 1 = x \vee 1$	[14843:17794]
17905,17904	$c(x) \vee 1 = 1$	[15828:17875,17796,16466]
17907,17906	$x \vee (y \vee 1) = 1$	[15822:17905]
17916,17915	$1 \vee (x \vee 0) = 1$	[17210:17875,17907,14,12354,13249]
17918,17917	$x \vee 1 = 1$	[15842:17916,12406,17905,17907]
17920,17919	$1 \wedge (x \vee 0) = x$	[5028:17916,17918,17916,17918,17918,14,12354]
17922,17921	$x \wedge 0 = 0$	[3877:17916,17918,14,17916,17918,17918,14,12354]
17980,17979	$(x \vee 0) \wedge 1 = x$	[16469:17918,17918,17918,14,17918,17918]
17989	$c(0 \vee x) = c(x \vee 0)$	[12896:17922,17918,17918,14,17920]
18409	$x \vee (c(x \vee 0) \vee (y \vee 0)) = 1$	[17573:17918]
20575	$(0 \vee x) \wedge 1 = x$	[17989 → 27,28,17980]
20873,20872	$x \vee 0 = x$	[13256 → 20575,17980]
21639	$x \vee (c(x) \vee y) = 1$	[18409:20873,20873]
21688,21687	$x \wedge 1 = x$	[17979:20873]
21862,21861	$c(c(x)) = x$	[27:20873,21688]
21863	$\$Ans(CC)$	[21861,2]
22975	$x \vee (x \wedge y) = x$	[21639 → 454,21862,21688,21862]
22977	$\$Ans(B1)$	[22975,1]

Independence of the $\mathcal{BA}$ 4-basis {AJ, DM, ONE, CUT} is open. In particular, we have not been able to find a proof or countermodel of

{AJ, DM, CUT} ⇒ ONE.

The simplest multiequation basis we know of for $\mathcal{BA}$ in terms of join and complement is the following, due to C. A. Meredith [13].

$c(c(x) \vee y) \vee x = x$	% MER_1
$c(c(x) \vee y) \vee (z \vee y) = y \vee (z \vee x)$	% MER_2

For comparison, the Robbins 3-basis for $\mathcal{BA}$ (in terms of join and complement) is the following [12].

$(x \vee y) \vee z = x \vee (y \vee z)$	% AJ2
$x \vee y = y \vee x$	% CJ
$c(c(x \vee c(y)) \vee c(x \vee y)) = x$	% Robbins

2.2 In Terms of the Sheffer Stroke

This section contain multiequation bases, in terms of the Sheffer stroke, for $\mathcal{OL}$, $\mathcal{OML}$, $\mathcal{MOL}$, and $\mathcal{BA}$. (There cannot be a basis for $\mathcal{L}$ in terms of the Sheffer stroke.) We prove each basis equivalent to the corresponding basis in terms of join, meet, and complement with two Otter jobs, and we show independence with Mace2 countermodels.

2.2.1 Ortholattices in Terms of the Sheffer Stroke

The following is a 3-basis for ortholattices in terms of the Sheffer stroke.

$(x|((y|z)|(y|z))) = (y|((x|z)|(x|z)))$ % A_SS
$((x|x)|(x|y)) = x$ % B_SS
$(x|(x|x)) = (y|(y|y))$ % ONE_SS

The following two Otter jobs show that this basis is definitionally equivalent to the (join/meet/complement) $\mathcal{OL}$ basis {AJ, B1, DM, CC, ONE}.

Proof OL-SS

1	$A \vee (B \vee C) \neq B \vee (A \vee C) \mid \$Ans(AJ)$	[]
2	$A \vee c(c(A) \vee c(B)) \neq A \mid \$Ans(B1_rewritten)$	[]
3	$A \wedge B \neq c(c(A) \vee c(B)) \mid \$Ans(DM)$	[]
4	$c(c(A)) \neq A \mid \$Ans(CC)$	[]
5	$A \vee c(A) \neq B \vee c(B) \mid \$Ans(ONE)$	[]
6	$(A\|B) \neq c(A) \vee c(B) \mid \$Ans(DEF_SS)$	[]
8	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[]
9	$((x\|x)\|(x\|y)) = x$	[]
11	$(x\|(x\|x)) = (y\|(y\|y))$	[]
12	$x \vee y = ((x\|x)\|(y\|y))$	[]
13	$((x\|x)\|(y\|y)) = x \vee y$	[12]
15	$x \wedge y = ((x\|y)\|(x\|y))$	[]
16	$((x\|y)\|(x\|y)) = x \wedge y$	[15]
18	$c(x) = (x\|x)$	[]
20,19	$(x\|x) = c(x)$	[18]
21	$(x\|c((y\|z))) = (y\|c((x\|z)))$	[8,20,20]
22	$(x\|c(x)) = (y\|c(y))$	[11,20,20]
24,23	$c((x\|y)) = x \wedge y$	[16:20]
25	$(c(x)\|c(y)) = x \vee y$	[13:20,20]
27	$(c(x)\|(x\|y)) = x$	[9:20]
29	$(x\|y \wedge z) = (y\|x \wedge z)$	[21,24,24]
37,36	$c(c(x)) = x$	[19 → 27,20]
38	$\$Ans(CC)$	[36,4]
41	$c(x) \wedge (x\|y) = c(x)$	[27 → 23]
47	$x \wedge c(x) = y \wedge c(y)$	[22 → 23,24]
49,48	$(x\|y) = c(x \wedge y)$	[23 → 36]
54	$c(x) \wedge c(x \wedge y) = c(x)$	[41:49]
58	$c(x \wedge (y \wedge z)) = c(y \wedge (x \wedge z))$	[29:49,49]
59	$c(c(x) \wedge c(y)) = x \vee y$	[25:49]
64	$c(x) \wedge x = y \wedge c(y)$	[36 → 47]
65	$x \wedge c(x) = c(y) \wedge y$	[64]
87	$x \wedge c(c(x) \wedge y) = x$	[36 → 54,37]
92,91	$x \wedge c(y \wedge c(y)) = x$	[64 → 54,37,37]
103	$c(x \wedge y) = c(y \wedge x)$	[91 → 58,92]
162	$x \wedge c(y \wedge c(x)) = x$	[103 → 87]
182	$x \wedge y = y \wedge x$	[103 → 36,37]
205,204	$c(x \wedge c(y)) = c(x) \vee y$	[36 → 59]
210,209	$c(c(x) \wedge y) = x \vee c(y)$	[36 → 59]
214	$x \vee c(x) = y \vee c(y)$	[65 → 59,210]
215	$\$Ans(ONE)$	[214,5]
218	$x \vee y = y \vee x$	[103 → 59,205,37]

223	$x \wedge (c(y) \vee x) = x$	[162:205]
330	$x \wedge (y \vee x) = x$	[36 → 223]
340	$x \wedge (x \vee y) = x$	[218 → 330]
344	$(x \vee y) \wedge y = y$	[182 → 330]
380	$(x \vee y) \wedge x = x$	[182 → 340]
386	$c(x \wedge (y \wedge (x \vee z))) = c(y \wedge x)$	[340 → 58]
400	$c((x \vee y) \wedge (z \wedge x)) = c(z \wedge x)$	[380 → 58]
913,912	$c(x \wedge y) = c(x) \vee c(y)$	[36 → 204]
914	$c(c(x) \vee y) \vee x = x$	[380 → 204,37]
916	$c(x \vee c(y)) \vee y = y$	[344 → 204,37]
1260	$c(x \vee y) \vee (c(z) \vee c(x)) = c(z) \vee c(x)$	[400:913,913,913]
1268	$c(x) \vee (c(y) \vee c(x \vee z)) = c(y) \vee c(x)$	[386:913,913,913]
1360	$(x\|y) = c(x) \vee c(y)$	[48:913]
1362	$\$Ans(DEF_SS)$	[1360,6]
1365	$x \vee c(c(x) \vee y) = x$	[218 → 914]
1367	$\$Ans(B1_rewritten)$	[1365,2]
1372	$c(x \vee y) \vee c(y) = c(y)$	[36 → 916]
1374	$x \vee c(y \vee c(x)) = x$	[218 → 916]
1461,1460	$c(x) \vee c(x \vee y) = c(x)$	[36 → 1365]
1465,1464	$c(x) \vee c(y \vee x) = c(x)$	[36 → 1374]
1810	$x \wedge y = c(c(x) \vee c(y))$	[912 → 36]
1812	$\$Ans(DM)$	[1810,3]
2611	$c((x \vee y) \vee z) \vee c(y) = c(y)$	[1464 → 1260,1465]
2613	$c((x \vee y) \vee z) \vee c(x) = c(x)$	[1460 → 1260,1461]
2631	$c(x) \vee (y \vee c(x \vee z)) = y \vee c(x)$	[36 → 1268,37]
2669	$((x \vee y) \vee z) \vee y = (x \vee y) \vee z$	[2611 → 1365,37]
3002,3001	$((x \vee y) \vee z) \vee x = (x \vee y) \vee z$	[2613 → 1365,37]
5126	$(x \vee y) \vee z = (y \vee x) \vee z$	[218 → 2669,3002]
5169	$x \vee (y \vee z) = (z \vee y) \vee x$	[218 → 5126]
14574	$(x \vee y) \vee (z \vee y) = z \vee (x \vee y)$	[1372 → 2631,37,37,37]
15295,15294	$(x \vee y) \vee (z \vee x) = y \vee (z \vee x)$	[5169 → 14574]
15296	$x \vee (y \vee z) = y \vee (x \vee z)$	[5126 → 14574,15295]
15297	$\$Ans(AJ)$	[15296,1]

Proof OL-SS-2

1	$(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C))) \ \mid \ \$Ans(SS)$	[]
2	$((A\|A)\|(A\|B)) \neq A \ \mid \ \$Ans(SS)$	[]
3	$(A\|(A\|A)) \neq (B\|(B\|B)) \ \mid \ \$Ans(SS)$	[]
4	$A \vee B \neq ((A\|A)\|(B\|B)) \ \mid \ \$Ans(J)$	[]
5	$A \wedge B \neq ((A\|B)\|(A\|B)) \ \mid \ \$Ans(M)$	[]
6	$c(A) \neq (A\|A) \ \mid \ \$Ans(C)$	[]
8	$x \vee (y \vee z) = y \vee (x \vee z)$	[]
9	$x \vee (x \wedge y) = x$	[]
12,11	$x \wedge y = c(c(x) \vee c(y))$	[]
14,13	$c(c(x)) = x$	[]
15	$x \vee c(x) = y \vee c(y)$	[]
16	$(x\|y) = c(x) \vee c(y)$	[]
18,17	$c(x) \vee c(y) = (x\|y)$	[16]
19	$x \vee c((x\|y)) = x$	[9:12,18]
21	$x \wedge y = c((x\|y))$	[11:18]
27	$c(x) \vee x = y \vee c(y)$	[13 → 15]

28	$x \vee c(x) = c(y) \vee y$	[27]
31	$x \vee c(y) = (c(x)\|y)$	[13 $\rightarrow$ 17]
39	$x \vee (y\|z) = c(y) \vee (x \vee c(z))$	[17 $\rightarrow$ 8]
40	$c(x) \vee (y \vee c(z)) = y \vee (x\|z)$	[39]
48	$(x\|c(x)) = c(y) \vee y$	[17 $\rightarrow$ 28]
49	$c(x) \vee x = (y\|c(y))$	[48]
75	$x \vee y = (c(x)\|c(y))$	[13 $\rightarrow$ 31]
77	$(c(x)\|(x\|y)) = x$	[19 $\rightarrow$ 31]
79	$(c(x)\|c(y)) = x \vee y$	[75]
86,85	$c(x) = (x\|x)$	[77 $\rightarrow$ 77,14]
87	$\$Ans(C)$	[85,6]
94,93	$x \vee y = ((x\|x)\|(y\|y))$	[79:86,86]
95	$\$Ans(J)$	[93,4]
97,96	$((x\|x)\|(x\|y)) = x$	[77:86]
98	$\$Ans(SS)$	[96,2]
100	$(x\|(x\|x)) = (y\|(y\|y))$	[49:86,94,97,86]
101	$\$Ans(SS)$	[100,3]
103	$(x\|(((y\|y)\|z)\|((y\|y)\|z))) = ((y\|y)\|((x\|z)\|(x\|z)))$	[40:86,86,94,97,94,97,94]
108	$x \wedge y = ((x\|y)\|(x\|y))$	[21:86]
110	$\$Ans(M)$	[108,5]
119	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[96 $\rightarrow$ 103,97,97]
120	$\$Ans(SS)$	[119,1]

The following three Mace2 jobs show that the $\mathcal{OL}$ 3-basis {A_SS, B_SS, ONE_SS} is independent.

Countermodel OL-SS-a

```
f(f(x,x),f(x,y)) = x.                              % B_SS
f(x,f(x,x)) = f(y,f(y,y)).                         % ONE_SS
f(A,f(f(B,C),f(B,C))) != f(B,f(f(A,C),f(A,C))).    % denial of A_SS
```

C: 0 A: 0 B: 1

f:	0	1	2
0	0	0	0
1	2	2	0
2	1	0	1

Table 18: OL-SS-a.out

Countermodel OL-SS-b

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(x,f(x,x)) = f(y,f(y,y)).                       % ONE_SS
f(f(A,A),f(A,B)) != A.                           % denial of B_SS
```

B: 0 A: 1

f:	0	1
0	0	0
1	0	0

Table 19: OL-SS-b.out

Countermodel OL-SS-c

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(f(x,x),f(x,y)) = x.                            % B_SS
f(A,f(A,A)) != f(B,f(B,B)).                      % denial of ONE_SS
```

B: 0 A: 1

f:	0	1	2
0	0	0	1
1	0	2	1
2	1	1	1

Table 20: OL-SS-c.out

2.2.2 Orthomodular Lattices in Terms of the Sheffer Stroke

The following is a 3-basis for orthomodular lattices in terms of the Sheffer stroke.

$(x|((y|z)|(y|z))) = (y|((x|z)|(x|z)))$ % A_SS
$((x|x)|(x|y)) = x$ % B_SS
$(x|(x|(x|y))) = (x|y)$ % OM_SS

The following two Otter jobs show that this basis is definitionally equivalent to the (join/meet/complement) $\mathcal{OML}$ basis {AJ, B1, DM, OM}.

Proof OML-SS

1	$A \vee (B \vee C) \neq B \vee (A \vee C) \mid \$Ans(AJ)$	[]
2	$A \vee (A \wedge B) \neq A \mid \$Ans(B1)$	[]
3	$A \wedge B \neq c(c(A) \vee c(B)) \mid \$Ans(DM)$	[]
4	$A \vee c(A \vee c(A \vee B)) \neq A \vee B \mid \$Ans(OM_rewritten)$	[]
5	$(A\|B) \neq c(A) \vee c(B) \mid \$Ans(DEF_SS)$	[]
7	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[]
8	$((x\|x)\|(x\|y)) = x$	[]
10	$(x\|(x\|(x\|y))) = (x\|y)$	[]
12	$x \vee y = ((x\|x)\|(y\|y))$	[]
13	$((x\|x)\|(y\|y)) = x \vee y$	[12]
15	$x \wedge y = ((x\|y)\|(x\|y))$	[]
16	$((x\|y)\|(x\|y)) = x \wedge y$	[15]
18	$c(x) = (x\|x)$	[]
20,19	$(x\|x) = c(x)$	[18]
21	$(x\|c((y\|z))) = (y\|c((x\|z)))$	[7,20,20]
23,22	$c((x\|y)) = x \wedge y$	[16:20]
25,24	$(c(x)\|c(y)) = x \vee y$	[13:20,20]
26	$(c(x)\|(x\|y)) = x$	[8:20]
28	$(x\|y \wedge z) = (y\|x \wedge z)$	[21,23,23]
39,38	$c(c(x)) = x \vee x$	[19 $\rightarrow$ 24]
44,43	$c(x) \wedge c(y) = c(x \vee y)$	[24 $\rightarrow$ 22]
45	$(c(x)\|(c(x)\|x \vee y)) = x \vee y$	[24 $\rightarrow$ 10,25]
49	$(x\|y) \vee (x\|y) = c(x \wedge y)$	[22 $\rightarrow$ 38]
64,63	$x \vee x = x$	[19 $\rightarrow$ 26,20,39]
74,73	$(x\|y) = c(x \wedge y)$	[49:64]
78,77	$c(c(x)) = x$	[38:64]
85	$x \vee (c(x) \wedge (x \vee y)) = x \vee y$	[45:74,74,44,78]

89	$x \vee (x \wedge y) = x$	[26:74,74,44,78]
91	$\$Ans(B1)$	[89,2]
100	$c(x \wedge (y \wedge z)) = c(y \wedge (x \wedge z))$	[28:74,74]
104,103	$c(x) \wedge y = c(x \vee c(y))$	[77 → 43]
107	$x \vee c(x \vee c(x \vee y)) = x \vee y$	[85:104]
109	$\$Ans(OM_rewritten)$	[107,4]
117,116	$x \wedge y = c(c(x) \vee c(y))$	[77 → 103]
118	$\$Ans(DM)$	[116,3]
125	$c(x) \vee (c(y) \vee c(z)) = c(y) \vee (c(x) \vee c(z))$	[100:117,117,78,78,117,117,78,78]
128	$(x\|y) = c(x) \vee c(y)$	[73:117,78]
130	$\$Ans(DEF_SS)$	[128,5]
143	$x \vee (c(y) \vee c(z)) = c(y) \vee (x \vee c(z))$	[77 → 125,78]
172	$x \vee (y \vee c(z)) = y \vee (x \vee c(z))$	[77 → 143,78]
209	$x \vee (y \vee z) = y \vee (x \vee z)$	[77 → 172,78]
210	$\$Ans(AJ)$	[209,1]

Proof OML-SS-2

1	$(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C))) \ \mid \ \$Ans(A_SS)$	[]
2	$((A\|A)\|(A\|B)) \neq A \ \mid \ \$Ans(B_SS)$	[]
3	$(A\|(A\|(A\|B))) \neq (A\|B) \ \mid \ \$Ans(OM_SS)$	[]
4	$A \vee B \neq ((A\|A)\|(B\|B)) \ \mid \ \$Ans(DEF_J)$	[]
5	$A \wedge B \neq ((A\|B)\|(A\|B)) \ \mid \ \$Ans(DEF_M)$	[]
6	$c(A) \neq (A\|A) \ \mid \ \$Ans(DEF_C)$	[]
8	$x \vee (y \vee z) = y \vee (x \vee z)$	[]
9	$x \vee (x \wedge y) = x$	[]
12,11	$x \wedge y = c(c(x) \vee c(y))$	[]
13	$x \vee (c(x) \wedge (x \vee y)) = x \vee y$	[]
14	$x \vee c(c(c(x)) \vee c(x \vee y)) = x \vee y$	[13,12]
16	$(x\|y) = c(x) \vee c(y)$	[]
18,17	$c(x) \vee c(y) = (x\|y)$	[16]
20,19	$x \vee c((x\|y)) = x$	[9:12,18]
21	$x \vee c((c(x)\|x \vee y)) = x \vee y$	[14:18]
23	$x \wedge y = c((x\|y))$	[11:18]
25	$x \vee (y \vee (z \vee u)) = z \vee (x \vee (y \vee u))$	[8 → 8]
27	$x \vee (y \vee c((x\|z))) = y \vee x$	[19 → 8]
29	$(x\|(c(x)\|y)) = c(x)$	[19 → 17]
31	$x \vee (y\|z) = c(y) \vee (x \vee c(z))$	[17 → 8]
32	$c(x) \vee (y \vee c(z)) = y \vee (x\|z)$	[31]
35	$x \vee c(c(x)) = x$	[29 → 19]
37	$x \vee c((c(x)\|x)) = x$	[19 → 21,20]
39	$(x\|(c(c(x))\|(x\|y))) = (x\|y)$	[17 → 21,18,18]
47	$x \vee (y \vee c(c(x))) = y \vee x$	[35 → 8]
53	$x \vee (y\|c(x)) = c(y) \vee x$	[17 → 47]
54	$c(x) \vee y = y \vee (x\|c(y))$	[53]
92,91	$c(x) \vee (y\|c(c(x))) = (y\|x)$	[17 → 54]
104	$c(x) \vee x = x \vee c(x)$	[37 → 27]
107,106	$c(x) \vee (x \vee y) = x \vee c(x)$	[21 → 27]
140	$x \vee (y \vee c(y)) = c(y) \vee (x \vee y)$	[104 → 8]
144	$c(x) \vee (y \vee x) = y \vee (x \vee c(x))$	[140]
149	$c(c(x)) \vee (x \vee c(x)) = (x\|c(x))$	[106 → 106,18]
156,155	$c(x) \vee (y \vee x) = x \vee c(x)$	[47 → 106]

170,169	$x \vee (y \vee c(y)) = y \vee c(y)$	[25 → 106,107]
171	$x \vee (c(x) \vee y) = x \vee c(x)$	[8 → 106]
174,173	$x \vee c(x) = (x\|c(x))$	[149:170]
176,175	$x \vee (y\|c(y)) = (y\|c(y))$	[144:156,174,174]
183	$x \vee (c(x) \vee y) = (x\|c(x))$	[171:174]
237	$(x\|c(x)) = (y\|c(y))$	[175 → 183,176]
252	$c(x) \vee (y\|z) = c(y) \vee (x\|z)$	[54 → 32,92]
270	$x \vee (y \vee c((z\|c(z)))) = y \vee x$	[237 → 27]
272,271	$x \vee c((y\|c(y))) = x$	[237 → 19]
274	$x \vee y = y \vee x$	[270,272]
279	$x \vee (y\|c(x)) = x \vee c(y)$	[54 → 274]
280	$c(x) \vee y = (x\|c(y)) \vee y$	[53 → 274]
283	$c(c(x)) \vee x = x$	[35 → 274]
294	$c((x\|y)) \vee x = x$	[19 → 274]
296	$(x\|y) = (y\|x)$	[17 → 274,18]
306	$x \vee (c((x\|y)) \vee z) = z \vee x$	[274 → 27]
314	$x \vee y = y \vee (c((y\|z)) \vee x)$	[306]
330	$(c(c(x))\|x) = c(x)$	[17 → 283]
352,351	$((c(x)\|y)\|x) = c(x)$	[29 → 296]
353	$x \vee (c(x)\|y) = c(y) \vee x$	[296 → 53]
354	$x \vee (y\|z) = c(z) \vee (x \vee c(y))$	[296 → 31]
363	$c(x) \vee y = y \vee (c(y)\|x)$	[353]
375	$c((x\|y)) \vee y = y$	[296 → 294]
378,377	$c((x\|c(x))) \vee y = y$	[237 → 294]
379	$x \vee ((x\|y)\|c(x)) = x$	[54 → 294]
409,408	$c(c(x)) = (c(x)\|c(x))$	[330 → 29]
472	$(x\|((c(x)\|c(x))\|(x\|y))) = (x\|y)$	[39:409]
480	$((x\|c(x))\|y) = c(y)$	[237 → 375,18]
493	$((x\|c(y))\|y) = c(y)$	[17 → 375]
497	$c(x) = ((y\|c(y))\|x)$	[480]
643	$c(((c((x\|c(x)))\|c((x\|c(x))))\|y)) = y$	[21 → 377,378,409,378]
766,765	$c(((x\|c(x))\|y)) = (c(y)\|c(y))$	[497 → 408]
898	$x \vee (c(x)\|(x\|y)) = x$	[296 → 379]
900	$((x\|y)\|c(x)) \vee x = x$	[274 → 379]
1132	$(c(x)\|(x\|y)) \vee x = x$	[274 → 898]
1162,1161	$((c(x)\|y)\|(c(x)\|c(x))) \vee (z\|x) = (z\|x)$	[900 → 32,18,409]
2136	$(c((x\|c(y)))\|c(y)) \vee (x\|c(y)) = (x\|c(y))$	[493 → 1132]
2138	$(c((c(x)\|y))\|c(x)) \vee (c(x)\|y) = (c(x)\|y)$	[351 → 1132]
3290	$x \vee (c(x)\|y) = x \vee c(y)$	[296 → 279]
4012	$(c(((c(x)\|y)\|(c(x)\|c(x))))\|x) = c(x)$	[379 → 363,409,409,1162]
4016	$(x\|c(y)) \vee y = y \vee (c(y)\|x)$	[280 → 363]
5787,5786	$((c((x\|c(x)))\|c((x\|c(x))))\|y) = ((x\|c(x))\|y)$	[377 → 3290,409,18]
5788	$(c(x)\|y) \vee x = x \vee c(y)$	[274 → 3290]
5806,5805	$(c(x)\|c(x)) = x$	[643:5787,766]
5807	$x \vee c(y) = (c(x)\|y) \vee x$	[5788]
5841,5840	$c(x) = (x\|x)$	[4012:5806,352,409,5806]
5842	$\$Ans(DEF_C)$	[5840,6]
5938	$(x\|(((x\|x)\|(x\|x))\|(x\|y))) = (x\|y)$	[472:5841,5841]
5963,5962	$((x\|x)\|y) \vee x = x \vee (y\|y)$	[5807,5841,5841]
5995,5994	$((x\|x)\|(x\|x)) = x$	[5805:5841,5841]
6607,6606	$(x\|(y\|y)) \vee y = y \vee ((y\|y)\|x)$	[4016:5841,5841]
7277,7276	$x \vee (y\|y) = ((x\|x)\|y)$	[2138:5841,5841,5841,5841,5963,5995,5963,5841]
7279,7278	$x \vee ((x\|x)\|y) = (y\|(x\|x))$	[2136:5841,5841,5841,5841,5963,5995,6607,5841]
7708,7707	$((x\|x)\|(x\|y)) = x$	[1132:5841,5963,7277]

7709	$\$Ans(B_SS)$	[7707,2]
7950,7949	$(x\|x) \vee y = (x\|(y\|y))$	[363:5841,5841,7279]
7957,7956	$x \vee (y\|z) = (z\|(((x\|x)\|y)\|((x\|x)\|y)))$	[354:5841,5841,7277,7950]
7961,7960	$x \vee y = ((x\|x)\|(y\|y))$	[314:5841,7950,7957,7708,7708]
7962	$\$Ans(DEF_J)$	[7960,4]
7967	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[252:5841,7961,5995,5841,7961,5995]
7968	$\$Ans(A_SS)$	[7967,1]
7977	$x \wedge y = ((x\|y)\|(x\|y))$	[23:5841]
7979	$\$Ans(DEF_M)$	[7977,5]
8024	$(x\|(x\|(x\|y))) = (x\|y)$	[5938:5995]
8026	$\$Ans(OM_SS)$	[8024,3]

The following three Mace2 jobs show that the $\mathcal{OML}$ 3-basis {A_SS, B_SS, OM_SS} is independent.

Countermodel OML-SS-a

```
f(f(x,x),f(x,y)) = x.                              % B_SS
f(x,f(x,f(x,y))) = f(x,y).                         % OM_SS
f(A,f(f(B,C),f(B,C))) != f(B,f(f(A,C),f(A,C))).  % denial of A_SS
```

C: 0 A: 0 B: 1

f:	0	1
0	0	0
1	1	1

Table 21: OML-SS-a.out

Countermodel OML-SS-b

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(x,f(x,f(x,y))) = f(x,y).                        % OM_SS
f(f(A,A),f(A,B)) != A.                            % denial of B_SS
```

B: 0 A: 1

f:	0	1
0	0	0
1	0	0

Table 22: OML-SS-b.out

Countermodel OML-SS-c

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(f(x,x),f(x,y)) = x.                             % B_SS
f(A,f(A,f(A,B))) != f(A,B).                       % denial of OM_SS
```

B: 0 A: 1

f:	0	1	2
0	2	2	2
1	2	1	1
2	2	1	0

Table 23: OML-SS-c.out

2.2.3 Modular Ortholattices in Terms of the Sheffer Stroke

The following is a 4-basis for modular ortholattices in terms of the Sheffer stroke.

$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	% A_SS
$((x\|x)\|(x\|y)) = x$	% B_SS
$(x\|(x\|x)) = (y\|(y\|y))$	% ONE_SS
$(x\|(y\|(x\|(z\|z)))) = (x\|(z\|(x\|(y\|y))))$	% MOD_SS

The following two Otter jobs show that this basis is definitionally equivalent to the (join/meet/complement) $\mathcal{MOL}$ basis {AJ, B1, DM, CC, ONE, MOD}.

Proof MOL-SS

1	$A \vee (B \vee C) \neq B \vee (A \vee C) \ \mid \ \$Ans(AJ)$	[]
2	$A \vee (A \wedge B) \neq A \ \mid \ \$Ans(B1)$	[]
3	$A \wedge B \neq c(c(A) \vee c(B)) \ \mid \ \$Ans(DM)$	[]
4	$c(c(A)) \neq A \ \mid \ \$Ans(CC)$	[]
5	$A \vee c(A) \neq B \vee c(B) \ \mid \ \$Ans(ONE)$	[]
6	$A \vee c(c(B) \vee c(A \vee C)) \neq A \vee c(c(C) \vee c(A \vee B)) \ \mid \ \$Ans(MOD_rewritten)$	[]
7	$(A\|B) \neq c(A) \vee c(B) \ \mid \ \$Ans(DEF_SS)$	[]
9	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[]
10	$((x\|x)\|(x\|y)) = x$	[]
12	$(x\|(x\|x)) = (y\|(y\|y))$	[]
13	$(x\|(y\|(x\|(z\|z)))) = (x\|(z\|(x\|(y\|y))))$	[]
14	$x \vee y = ((x\|x)\|(y\|y))$	[]
15	$((x\|x)\|(y\|y)) = x \vee y$	[14]
17	$x \wedge y = ((x\|y)\|(x\|y))$	[]
18	$((x\|y)\|(x\|y)) = x \wedge y$	[17]
20	$c(x) = (x\|x)$	[]
22,21	$(x\|x) = c(x)$	[20]
23	$(x\|c((y\|z))) = (y\|c((x\|z)))$	[9,22,22]
24	$(x\|c(x)) = (y\|c(y))$	[12,22,22]
25	$(x\|(y\|(x\|c(z)))) = (x\|(z\|(x\|c(y))))$	[13,22,22]
27,26	$c((x\|y)) = x \wedge y$	[18:22]
28	$(c(x)\|c(y)) = x \vee y$	[15:22,22]
30	$(c(x)\|(x\|y)) = x$	[10:22]
32	$(x\|y \wedge z) = (y\|x \wedge z)$	[23,27,27]
40,39	$c(c(x)) = x$	[21 → 30,22]
41	$\$Ans(CC)$	[39,4]
44	$c(x) \wedge (x\|y) = c(x)$	[30 → 26]
50	$x \wedge c(x) = y \wedge c(y)$	[24 → 26,27]
52,51	$(x\|y) = c(x \wedge y)$	[26 → 39]
57	$c(x) \wedge c(x \wedge y) = c(x)$	[44:52]
61	$c(x \wedge (y \wedge z)) = c(y \wedge (x \wedge z))$	[32:52,52]
62	$c(c(x) \wedge c(y)) = x \vee y$	[28:52]
64	$c(x \wedge c(y \wedge c(x \wedge c(z)))) = c(x \wedge c(z \wedge c(x \wedge c(y))))$	[25:52,52,52,52,52,52]
68	$c(x) \wedge x = y \wedge c(y)$	[39 → 50]
69	$x \wedge c(x) = c(y) \wedge y$	[68]
91	$x \wedge c(c(x) \wedge y) = x$	[39 → 57,40]
96,95	$x \wedge c(y \wedge c(y)) = x$	[68 → 57,40,40]
107	$c(x \wedge y) = c(y \wedge x)$	[95 → 61,96]
166	$x \wedge c(y \wedge c(x)) = x$	[107 → 91]
186	$x \wedge y = y \wedge x$	[107 → 39,40]

209,208	$c(x \wedge c(y)) = c(x) \vee y$	[39 → 62]
214,213	$c(c(x) \wedge y) = x \vee c(y)$	[39 → 62]
215	$x \vee (x \wedge y) = x$	[91 → 62,40,40]
217	$\$Ans(B1)$	[215,2]
219	$x \vee c(x) = y \vee c(y)$	[69 → 62,214]
220	$\$Ans(ONE)$	[219,5]
223	$x \vee y = y \vee x$	[107 → 62,209,40]
228	$x \wedge (c(y) \vee x) = x$	[166:209]
240	$c(x) \vee (y \wedge (c(x) \vee z)) = c(x) \vee (z \wedge (c(x) \vee y))$	[64:209,209,209,209]
336	$x \wedge (y \vee x) = x$	[39 → 228]
346	$x \wedge (x \vee y) = x$	[223 → 336]
350	$(x \vee y) \wedge y = y$	[186 → 336]
386	$(x \vee y) \wedge x = x$	[186 → 346]
392	$c(x \wedge (y \wedge (x \vee z))) = c(y \wedge x)$	[346 → 61]
406	$c((x \vee y) \wedge (z \wedge x)) = c(z \wedge x)$	[386 → 61]
919,918	$c(x \wedge y) = c(x) \vee c(y)$	[39 → 208]
920	$c(c(x) \vee y) \vee x = x$	[386 → 208,40]
922	$c(x \vee c(y)) \vee y = y$	[350 → 208,40]
1266	$c(x \vee y) \vee (c(z) \vee c(x)) = c(z) \vee c(x)$	[406:919,919,919]
1274	$c(x) \vee (c(y) \vee c(x \vee z)) = c(y) \vee c(x)$	[392:919,919,919]
1366	$(x\|y) = c(x) \vee c(y)$	[51:919]
1368	$\$Ans(DEF_SS)$	[1366,7]
1371	$x \vee c(c(x) \vee y) = x$	[223 → 920]
1377	$c(x \vee y) \vee c(y) = c(y)$	[39 → 922]
1379	$x \vee c(y \vee c(x)) = x$	[223 → 922]
1466,1465	$c(x) \vee c(x \vee y) = c(x)$	[39 → 1371]
1470,1469	$c(x) \vee c(y \vee x) = c(x)$	[39 → 1379]
1816,1815	$x \wedge y = c(c(x) \vee c(y))$	[918 → 39]
1817	$\$Ans(DM)$	[1815,3]
2100	$c(x) \vee c(c(y) \vee c(c(x) \vee z)) = c(x) \vee c(c(z) \vee c(c(x) \vee y))$	[240:1816,1816]
2617	$c((x \vee y) \vee z) \vee c(y) = c(y)$	[1469 → 1266,1470]
2619	$c((x \vee y) \vee z) \vee c(x) = c(x)$	[1465 → 1266,1466]
2637	$c(x) \vee (y \vee c(x \vee z)) = y \vee c(x)$	[39 → 1274,40]
2675	$((x \vee y) \vee z) \vee y = (x \vee y) \vee z$	[2617 → 1371,40]
3006,3005	$((x \vee y) \vee z) \vee x = (x \vee y) \vee z$	[2619 → 1371,40]
5132	$(x \vee y) \vee z = (y \vee x) \vee z$	[223 → 2675,3006]
5175	$x \vee (y \vee z) = (z \vee y) \vee x$	[223 → 5132]
7211	$x \vee c(c(y) \vee c(x \vee z)) = x \vee c(c(z) \vee c(x \vee y))$	[39 → 2100,40,40,40]
7212	$\$Ans(MOD_rewritten)$	[7211,6]
14692	$(x \vee y) \vee (z \vee y) = z \vee (x \vee y)$	[1377 → 2637,40,40,40]
15413,15412	$(x \vee y) \vee (z \vee x) = y \vee (z \vee x)$	[5175 → 14692]
15414	$x \vee (y \vee z) = y \vee (x \vee z)$	[5132 → 14692,15413]
15415	$\$Ans(AJ)$	[15414,1]

Proof MOL-SS-2

1	$(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C)))$ | $\$Ans(A_SS)$	[]
2	$((A\|A)\|(A\|B)) \neq A$ | $\$Ans(B_SS)$	[]
3	$(A\|(A\|A)) \neq (B\|(B\|B))$ | $\$Ans(ONE_SS)$	[]
4	$(A\|(B\|(A\|(C\|C)))) \neq (A\|(C\|(A\|(B\|B))))$ | $\$Ans(MOD_SS)$	[]
5	$A \vee B \neq ((A\|A)\|(B\|B))$ | $\$Ans(DEF_J)$	[]
6	$A \wedge B \neq ((A\|B)\|(A\|B))$ | $\$Ans(DEF_M)$	[]

7	$c(A) \neq (A\|A) \ \mid \ \$Ans(DEF_C)$	[]
9	$x \vee (y \vee z) = y \vee (x \vee z)$	[]
10	$x \vee (x \wedge y) = x$	[]
13,12	$x \wedge y = c(c(x) \vee c(y))$	[]
15,14	$c(c(x)) = x$	[]
16	$x \vee c(x) = y \vee c(y)$	[]
17	$x \vee (y \wedge (x \vee z)) = x \vee (z \wedge (x \vee y))$	[]
18	$x \vee c(c(y) \vee c(x \vee z)) = x \vee c(c(z) \vee c(x \vee y))$	[17,13,13]
19	$(x\|y) = c(x) \vee c(y)$	[]
21,20	$c(x) \vee c(y) = (x\|y)$	[19]
22	$x \vee c((x\|y)) = x$	[10:13,21]
24	$x \vee c((y\|x \vee z)) = x \vee c((z\|x \vee y))$	[18,21,21]
25	$x \wedge y = c((x\|y))$	[12:21]
31	$c(x) \vee x = y \vee c(y)$	[14 → 16]
32	$x \vee c(x) = c(y) \vee y$	[31]
35	$x \vee c(y) = (c(x)\|y)$	[14 → 20]
37	$(x\|(c(x)\|y)) = c(x)$	[22 → 20]
43	$x \vee (y\|z) = c(y) \vee (x \vee c(z))$	[20 → 9]
44	$c(x) \vee (y \vee c(z)) = y \vee (x\|z)$	[43]
52	$(x\|c(x)) = c(y) \vee y$	[20 → 32]
53	$c(x) \vee x = (y\|c(y))$	[52]
60	$(x\|(y\|(x\|z))) = (x\|(c(z)\|c(x) \vee y))$	[20 → 24,21,21]
65	$(x\|(y\|c(x) \vee z)) = (x\|(z\|c(x) \vee y))$	[20 → 24,21]
68	$(x\|(c(y)\|c(x) \vee z)) = (x\|(z\|(x\|y)))$	[60]
73	$x \vee y = (c(x)\|c(y))$	[14 → 35]
77	$(c(x)\|(x\|y)) = x$	[22 → 35]
79	$(c(x)\|c(y)) = x \vee y$	[73]
93,92	$c(x) = (x\|x)$	[77 → 77,15]
94	$\$Ans(DEF_C)$	[92,7]
100,99	$x \vee y = ((x\|x)\|(y\|y))$	[79:93,93]
101	$\$Ans(DEF_J)$	[99,5]
103,102	$((x\|x)\|(x\|y)) = x$	[77:93]
104	$\$Ans(B_SS)$	[102,2]
106	$(x\|((y\|y)\|(x\|(z\|z)))) = (x\|(z\|(x\|y)))$	[68:93,93,100,103]
108	$(x\|(y\|(x\|(z\|z)))) = (x\|(z\|(x\|(y\|y))))$	[65:93,100,103,93,100,103]
109	$\$Ans(MOD_SS)$	[108,4]
112	$(x\|(x\|x)) = (y\|(y\|y))$	[53:93,100,103,93]
113	$\$Ans(ONE_SS)$	[112,3]
115	$(x\|(((y\|y)\|z)\|((y\|y)\|z))) = ((y\|y)\|((x\|z)\|(x\|z)))$	[44:93,93,100,103,100,103,100]
119,118	$(x\|((x\|x)\|y)) = (x\|x)$	[37:93,93]
120	$x \wedge y = ((x\|y)\|(x\|y))$	[25:93]
122	$\$Ans(DEF_M)$	[120,6]
145	$((x\|x)\|(y\|x)) = x$	[106 → 108,103,103,103,119,103]
176	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[145 → 115,103,103]
177	$\$Ans(A_SS)$	[176,1]

The following four Mace2 jobs show that $\mathcal{MOL}$ 4-basis {A_SS, B_SS, ONE_SS, MOD_SS} is independent.

Countermodel MOL-SS-a

```
f(f(x,x),f(x,y)) = x.                          % B_SS
f(x,f(x,x)) = f(y,f(y,y)).                     % ONE_SS
f(x,f(y,f(x,f(z,z)))) = f(x,f(z,f(x,f(y,y)))).  % MOD_SS
```

```
f(A,f(f(B,C),f(B,C))) != f(B,f(f(A,C),f(A,C))).  % denial of A_SS
```

C: 0 A: 0 B: 1

f:	0	1	2
0	0	0	0
1	2	2	0
2	1	0	1

Table 24: MOL-SS-a.out

Countermodel MOL-SS-b

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(x,f(x,x)) = f(y,f(y,y)).                       % ONE_SS
f(x,f(y,f(x,f(z,z)))) = f(x,f(z,f(x,f(y,y)))).   % MOD_SS
f(f(A,A),f(A,B)) != A.                           % denial of B_SS
```

B: 0 A: 1

f:	0	1
0	0	0
1	0	0

Table 25: MOL-SS-b.out

Countermodel MOL-SS-c

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(f(x,x),f(x,y)) = x.                            % B_SS
f(x,f(y,f(x,f(z,z)))) = f(x,f(z,f(x,f(y,y)))).   % MOD_SS
f(A,f(A,A)) != f(B,f(B,B)).                      % denial of ONE_SS
```

B: 0 A: 1

f:	0	1	2
0	0	0	1
1	0	2	1
2	1	1	1

Table 26: MOL-SS-c.out

Countermodel MOL-SS-d

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(f(x,x),f(x,y)) = x.                            % B_SS
f(x,f(x,x)) = f(y,f(y,y)).                       % ONE_SS
f(A,f(B,f(A,f(C,C)))) != f(A,f(C,f(A,f(B,B)))).  % denial of MOD_SS
```

B: 0 A: 0 C: 1

f:	0	1	2	3	4	5
0	2	3	3	2	3	1
1	3	5	0	5	3	3
2	3	0	0	0	3	3
3	2	5	0	4	3	1
4	3	3	3	3	3	3
5	1	3	3	1	3	1

Table 27: MOL-SS-d.out

2.2.4 Boolean Algebra in Terms of the Sheffer Stroke

The following is a 2-basis for Boolean algebra in terms of the Sheffer stroke.

$(x|((y|z)|(y|z))) = (y|((x|z)|(x|z)))$ % A_SS
$((x|(y|y))|(x|y)) = x$ % CUT_SS

The two Otter job below show that this basis is definitionally equivalent to the (join/meet/complement) $\mathcal{BA}$ basis {AJ, DM, ONE, CUT}.

For reference, the simplest multiequation basis for $\mathcal{BA}$ in terms of the Sheffer stroke is known to be the following [20].

$x|y = y|x$
$(x|y)|(x|(y|z)) = x$

Proof BA-SS

1	$A \vee (B \vee C) \neq B \vee (A \vee C) \mid \$Ans(AJ)$	[]
2	$A \wedge B \neq c(c(A) \vee c(B)) \mid \$Ans(DM)$	[]
3	$A \vee c(A) \neq B \vee c(B) \mid \$Ans(ONE)$	[]
4	$c(c(A \vee c(B)) \vee c(A \vee B)) \neq A \mid \$Ans(CUT_rewritten)$	[]
5	$(A\|B) \neq c(A) \vee c(B) \mid \$Ans(DEF_SS)$	[]
7	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[]
8	$((x\|(y\|y))\|(x\|y)) = x$	[]
10	$x \vee y = ((x\|x)\|(y\|y))$	[]
11	$((x\|x)\|(y\|y)) = x \vee y$	[10]
13	$x \wedge y = ((x\|y)\|(x\|y))$	[]
14	$((x\|y)\|(x\|y)) = x \wedge y$	[13]
16	$c(x) = (x\|x)$	[]
18,17	$(x\|x) = c(x)$	[16]
19	$(x\|c((y\|z))) = (y\|c((x\|z)))$	[7,18,18]
21,20	$c((x\|y)) = x \wedge y$	[14:18]
23,22	$(c(x)\|c(y)) = x \vee y$	[11:18,18]
24	$((x\|c(y))\|(x\|y)) = x$	[8:18]
26	$(x\|y \wedge z) = (y\|x \wedge z)$	[19,21,21]
27	$x \wedge x = c(c(x))$	[17 → 20]
30,29	$(x \wedge y\|c(z)) = (x\|y) \vee z$	[20 → 22]
31	$(c(x)\|y \wedge z) = x \vee (y\|z)$	[20 → 22]
33,32	$c(c(x)) = x \vee x$	[17 → 22]
36,35	$x \wedge x = x \vee x$	[27:33]
38,37	$c(x) \wedge c(y) = c(x \vee y)$	[22 → 20]
39	$((x\|y \wedge z)\|(x\|(y\|z))) = x$	[20 → 24]
41	$(x \vee y\|(c(x)\|y)) = c(x)$	[22 → 24]
43	$(((x\|c(y))\|x \wedge y)\|x) = (x\|c(y))$	[24 → 24,21]
45	$((c(x)\|y \vee y)\|x \vee y) = c(x)$	[22 → 24,33]
47	$((x\|c(x))\|c(x)) = x$	[17 → 24]
49	$(x\|c(y)) \wedge (x\|y) = c(x)$	[24 → 20]
52,51	$c(x \vee x) = c(x) \vee c(x)$	[32 → 32]
54,53	$(x\|y) \vee (x\|y) = c(x \wedge y)$	[20 → 32]
55	$((x\|y \vee y)\|(x\|c(y))) = x$	[32 → 24]
58,57	$(c(x)\|y \vee y) = x \vee c(y)$	[32 → 22]
60,59	$(x \vee x\|c(y)) = c(x) \vee y$	[32 → 22]
62,61	$(x \vee c(y)\|x \vee y) = c(x)$	[45:58]

63	$(((x\|y)\|x \wedge y)\|x \wedge y) = (x\|y)$	[20 → 47,21]
69	$(x\|c(x)) \wedge c(x) = c(x)$	[47 → 20]
71	$(x\|y \vee y) = (y\|x \wedge y)$	[35 → 26]
72	$(x\|(x \wedge y) \wedge y) = c(x \wedge y)$	[17 → 26]
74	$(x\|y \wedge x) = (y\|x \vee x)$	[71]
75	$((x\|c(y \wedge z))\|(y\|x \wedge z)) = x$	[26 → 24]
77	$x \wedge (y \wedge z) = y \wedge (x \wedge z)$	[26 → 20,21]
79,78	$(x \vee x) \wedge c(y) = c(c(x) \vee y)$	[32 → 37]
82,81	$c(x) \wedge (y \vee y) = c(x \vee c(y))$	[32 → 37]
94	$((x\|y)\|x \wedge y) \wedge (x \wedge y) = x \wedge y$	[20 → 69,21,21]
96	$(x\|c(y)) = ((y\|c(y))\|x \wedge c(y))$	[69 → 26]
102	$(x \wedge y\|z \wedge u) = (x\|y) \vee (z\|u)$	[20 → 29]
118	$(x \vee y) \wedge (c(x)\|y) = x \vee x$	[41 → 20,33]
122	$(x\|y \vee y) \wedge (x\|c(y)) = c(x)$	[32 → 49]
132	$(x \vee y\|c(x) \wedge y) \wedge c(x) = c(x \vee y)$	[41 → 49,21]
141,140	$(x \vee c(y)) \wedge (x \vee y) = x \vee x$	[22 → 49,33,58,33]
147,146	$(x \vee x\|y \vee y) = c(x) \vee c(y)$	[32 → 57]
155	$(x \vee x\|y \wedge z) = c(x) \vee (y\|z)$	[32 → 31]
157,156	$(x\|c(y) \wedge z) = y \vee (x\|z)$	[26 → 31]
161,160	$(x \vee (x \vee y\|y)) \wedge c(x) = c(x \vee y)$	[132:157]
176	$(x\|(x \vee c(x)) \wedge x) = c(x)$	[61 → 71]
184	$x \wedge (y \wedge x) = y \wedge (x \vee x)$	[71 → 20,21]
185	$x \wedge (y \vee y) = y \wedge (x \wedge y)$	[184]
200	$x \vee (y \vee x) = y \vee (x \vee x)$	[37 → 74,23,58,33]
203	$x \vee (y \vee y) = y \vee (x \vee y)$	[200]
214	$((x\|c(y \wedge x))\|(y\|x \vee x)) = x$	[74 → 24]
218	$x \wedge (y \wedge (z \wedge u)) = z \wedge (x \wedge (y \wedge u))$	[77 → 77]
223	$x \wedge c(y \vee z) = c(y) \wedge (x \wedge c(z))$	[37 → 77]
226	$x \wedge (y \wedge (z \wedge u)) = y \wedge (z \wedge (x \wedge u))$	[218]
246,245	$x \wedge ((x \vee c(x)) \wedge x) = x \vee x$	[176 → 20,33]
250,249	$(c(x)\|(x\|(x \vee c(x)\|x))) = x$	[176 → 39]
251	$((x\|y \vee y)\|(y\|(x\|y))) = y$	[74 → 39]
253	$((x\|y \wedge z)\|(y\|(x\|z))) = y$	[26 → 39]
279	$(x \wedge y\|x \wedge (y \vee y)) = (y\|(x \wedge y) \vee (x \wedge y))$	[184 → 74]
331,330	$(x \vee (y \vee x)\|y \vee c(x)) = c(y)$	[203 → 41,58]
338,337	$(x\|y \wedge x) \vee (y\|x \vee x) = c(y \wedge (x \vee x))$	[71 → 53]
342	$c(x \wedge (y \vee y)) = c(y \wedge (x \wedge y))$	[74 → 53,338]
348	$((x\|y \wedge x)\|(y\|c(x))) = y$	[71 → 55]
357,356	$((x\|(x\|y) \wedge y)\|x \wedge y) = (x\|y)$	[26 → 63]
367,366	$(x \vee y) \vee (x \vee y) = x \vee ((x \vee y) \vee y)$	[37 → 72,38,23,38,33]
373,372	$x \vee (c(x) \wedge y\|y) = c(c(x) \wedge y)$	[31 → 72]
391,390	$(x\|c((x \wedge y) \wedge y)) \wedge c(x \wedge y) = c(x)$	[72 → 49]
392	$((x\|c((x \wedge y) \wedge y))\|c(x \wedge y)) = x$	[72 → 24]
400	$(x \vee x) \wedge (y \wedge z) = c(c(x) \vee (y\|z))$	[20 → 78]
401	$c(c(x) \vee (y\|z)) = (x \vee x) \wedge (y \wedge z)$	[400]
409	$x \wedge (c(y) \wedge x) = c(y \vee c(x))$	[185 → 81]
420	$(x \vee (y \vee x)) \wedge (y \vee c(x)) = y \vee y$	[203 → 118,58]
434	$((c(x)\|y)\|x \vee x) = (x \vee y\|c(c(x) \wedge y))$	[118 → 74,54]
466,465	$(x\|c(y \vee z)) = y \vee (x\|c(z))$	[37 → 156]
468,467	$x \vee (x \vee ((c(x)\|y)\|y)\|y) = (c(x)\|y)$	[63 → 156,157]
469	$x \vee (c(y)\|z) = y \vee (c(x)\|z)$	[31 → 156]
479	$(x \vee (y\|z)\|(y\|(c(x)\|z))) = y$	[156 → 39]
486	$c(x \vee (y\|z)) = y \wedge (c(x) \wedge z)$	[156 → 20]
488	$x \wedge (c(y) \wedge z) = c(y \vee (x\|z))$	[486]

515	$(x\|y \wedge x) \wedge (y\|c(x)) = c(y)$	[348 → 20]
548	$((x\|y \vee y)\|((y\|c(y))\|x \wedge c(y))) = x$	[69 → 75,33]
641,640	$((x\|c(y))\|c(x)) = ((x\|y \vee y)\|c(x \wedge c(y)))$	[122 → 74,54]
648,647	$((x\|x \vee x)\|c(x \wedge c(x))) = x$	[47:641]
662	$(x\|(x\|y) \wedge y) \wedge (x \wedge y) = x \wedge y$	[26 → 94]
684	$c(x) \wedge (x\|(x \vee c(x)\|x)) = c(x)$	[249 → 20]
688	$(x \vee c(y)\|(y\|(c(x)\|y))) = y$	[57 → 251]
700	$(x\|y \vee y) \wedge (y\|(x\|y)) = c(y)$	[251 → 20]
704	$x \vee ((x\|c(x))\|c(y)) = x \vee y$	[43 → 96,23,23,157,161,466]
790	$((c(x)\|(x \vee y) \wedge y)\|c(x)) = x \vee y$	[41 → 253]
803,802	$(x\|y \wedge z) \wedge (y\|(x\|z)) = c(y)$	[253 → 20]
845	$(c(x) \vee (y \vee c(x))\|y \vee (x \vee x)) = c(y)$	[32 → 330]
868	$(x \wedge (y \wedge x)) \vee z = (y \wedge (x \vee x)) \vee z$	[342 → 22,23]
927	$x \vee (y \vee z) = y \vee (x \vee z)$	[22 → 465,23]
928	$\$Ans(AJ)$	[927,1]
1154	$x \vee ((x\|c(x))\|y \vee y) = x \vee c(y)$	[32 → 704]
1156	$x \vee ((x\|c(x))\|y \wedge z) = x \vee (y\|z)$	[20 → 704]
1163	$x \vee (y\|z) = x \vee ((x\|c(x))\|y \wedge z)$	[1156]
1166	$(x\|c(x)) \wedge c(x \vee y) = c(x \vee y)$	[704 → 486,38]
1241,1240	$(x\|y \vee ((y \vee y) \vee y)) = c(y) \vee (x\|y \vee y)$	[74 → 155,367]
1291	$((x\|c(c(y) \wedge x))\|y \vee c(x)) = x$	[57 → 214]
1384	$(c(x) \vee (y \vee c(x))) \wedge (y \vee (x \vee x)) = y \vee y$	[32 → 420]
1456,1455	$c(x) \wedge (y \wedge (x\|(x \vee c(x)\|x))) = y \wedge c(x)$	[684 → 77]
1477	$(c(x) \vee c(y)) \wedge (y\|(x \vee x\|y)) = c(y)$	[146 → 700]
2081	$((x\|y) \vee (x \wedge y)\|c(x)) = x$	[96 → 392,30,391]
2100	$(x \vee c(x)\|y \wedge (x \wedge z)) = (y\|x \wedge z)$	[253 → 2081,803,21]
2105	$(c(x) \vee (x \vee x)\|c(x)) = x$	[176 → 2081,246]
2119	$((x\|y \vee y) \vee (y \wedge (x \wedge y))\|c(y)) = y$	[74 → 2081]
2137	$((x\|y \wedge z) \vee (y \wedge (x \wedge z))\|c(y)) = y$	[26 → 2081]
2198,2197	$x \vee ((c(x) \wedge y) \vee c(c(x) \wedge y)\|y) = (c(x)\|y)$	[469 → 2105,21,21,373,21,157]
2648	$c(x \vee (y \vee x)) \vee z = c(y \vee (x \vee x)) \vee z$	[37 → 868,38,82,33]
2652	$c(x \vee (y \vee y)) \vee z = c(y \vee (x \vee y)) \vee z$	[2648]
2817	$x \vee (y\|(x\|c(x)) \wedge y) = x \vee c(y)$	[71 → 1154]
2935	$(x\|y) \vee (x\|y \vee y) = (y\|(x \wedge y) \vee (x \wedge y))$	[102 → 279]
2936	$(x\|(y \wedge x) \vee (y \wedge x)) = (y\|x) \vee (y\|x \vee x)$	[2935]
2981	$(x \vee x) \wedge (y \wedge ((x \vee c(x)) \wedge y)) = c(c(x) \vee c(y))$	[2817 → 401,33,58]
3112,3111	$x \vee (y \vee ((x\|c(x))\|z)) = y \vee (c(x)\|z)$	[469 → 1163,157]
3169,3168	$(x \vee c(x)\|x \wedge y) = (x\|y)$	[662 → 2100,357]
3201,3200	$(x\|(x \vee c(x)) \wedge y) = (x\|y)$	[26 → 3168]
3212	$(x\|y) \wedge (x\|(x \vee c(x)\|y)) = c(x)$	[3168 → 802]
3217,3216	$((x\|y)\|(x\|(x \vee c(x)\|y))) = x$	[3168 → 253]
3228	$(x \vee c(x)) \wedge (x \wedge y) = x \wedge y$	[3168 → 20,21]
3264	$x \wedge ((x \vee c(x)) \wedge y) = x \wedge y$	[3200 → 20,21]
3283,3282	$x \wedge (y \wedge ((x \vee c(x)) \wedge z)) = x \wedge (y \wedge z)$	[226 → 3228]
3290	$((x \vee c(x)) \vee c(x \vee c(x))\|x \wedge y) = (x\|y)$	[3228 → 3168,3169]
3298	$(x\|c(x)) \wedge (c(x) \wedge y) = c(x) \wedge y$	[467 → 1166,21,468,21]
3306	$c(x) \wedge (c(x)\|c(x) \vee x) = c(x \vee c(x))$	[1477 → 3264,33,82,60]
3312	$(x\|((x \vee c(x)) \vee c(x \vee c(x))) \wedge y) = (x\|y)$	[3264 → 3200,3201]
3390,3389	$c(x \vee ((x\|c(x))\|y)) = c(x) \wedge y$	[488 → 3298]
3406,3405	$(x \vee ((x\|c(x))\|y)) \vee z = (c(x)\|y) \vee z$	[3298 → 29,30,157]
3430	$x \vee ((x\|c(x))\|y) = (c(x)\|y)$	[3389 → 2105,3390,367,3406,3112,54,157,2198]
3432	$c(x) \vee (x \vee c(x)\|y) = (x \vee x\|y)$	[32 → 3430,58,33]
3538	$(x \vee x\|(x\|(c(x)\|x))) = c(x) \vee x$	[688 → 3432]
3580	$(x \vee (x \vee x)\|(x \vee x) \vee c(x)) = c(x) \vee c(x)$	[3538 → 479,33,60,62,33,33]

3593,3592	$((x\|c(y)) \vee (x \wedge c(y))\|y \vee y) = c(y)$	[684 → 2137,1456,33]
3650	$(c(x)\|x) = (x\|c(x)) \vee (x \wedge c(x))$	[3200 → 548,21,3593,641,648,21]
3663,3662	$(x \vee c(x)) \vee c(x \vee c(x)) = c(x) \vee x$	[32 → 3650,60,33,58,33,82]
3664	$(x\|(c(x) \vee x) \wedge y) = (x\|y)$	[3312:3663]
3668	$(c(x) \vee x\|x \wedge y) = (x\|y)$	[3290:3663]
3683	$c(x) \vee (c(x) \wedge x) = c(x)$	[3650 → 434,3593,60]
3692	$(x \vee x) \vee c(c(x) \vee x) = x \vee x$	[32 → 3683,33,79,33]
3748,3747	$(c(x)\|((x \vee x) \vee c(x)) \wedge y) = (c(x)\|y)$	[32 → 3664]
3788,3787	$(c(x) \vee x\|x \vee x) = c(x)$	[74 → 3664,18]
3809	$c(x) \wedge (c(x) \vee x\|c(x)) = c(c(x) \vee x)$	[3664 → 515,18]
3871	$(c(x)\|(x\|(c(x) \vee x\|x))) = x$	[3787 → 251]
3955	$((x \wedge y) \vee (x\|y)\|c(x)) = x$	[3212 → 3668,21,3217]
3976,3975	$((x \vee x) \vee c(x)\|c(x)) = x$	[684 → 3668,33,250]
4022	$(((x \vee x) \vee c(x)\|x \vee x)\|x) = (x \vee x) \vee c(x)$	[3975 → 55]
4029,4028	$(x\|((x \vee x) \vee c(x)\|x)) = (x \vee x) \vee c(x)$	[3975 → 24]
4115	$((c(x) \wedge (y \wedge c(z))) \vee (x \vee (y\|c(z)))\|c(y)) = y$	[223 → 3955,466]
4192,4191	$c(x) \vee x = (x \vee x) \vee c(x)$	[3692 → 1291,52,141,52,33,33,367,1241,3788,147,33]
4196,4195	$c(x) \wedge x = c((x \vee x) \vee c(x))$	[3809:4192,3976,4192]
4214,4213	$(c(x)\|(x \vee x) \vee c(x)) = x$	[3871:4192,4029]
4236,4235	$((x \vee x) \vee c(x)\|x \vee x) = c(x)$	[3787:4192]
4252,4251	$(x \vee c(x)) \vee c(x \vee c(x)) = (x \vee x) \vee c(x)$	[3662:4192]
4274,4273	$c((x \vee x) \vee c(x)) = c(x \vee c(x))$	[3306:4192,4214,4196]
4300,4299	$(c(x)\|x) = (x \vee x) \vee c(x)$	[4022:4236]
4401	$c(x) \wedge x = c(x \vee c(x))$	[4195:4274]
4446,4445	$x \wedge c(x \vee c(x)) = c(x \vee c(x))$	[4401 → 409]
4501,4500	$(x \vee x) \vee c(x) = x \vee c(x)$	[4401 → 356,4300,3748,4300,466,33,4236,4300]
4503,4502	$(x \vee c(x)\|c(x)) = x$	[4401 → 2119,58,4446,4252,4501]
4512	$(x \vee (c(x) \vee c(x))\|c(x)) = x \vee x$	[4401 → 43,18,33,466,33,18,52,18,33]
4556,4555	$c(x) \vee x = x \vee c(x)$	[4191:4501]
4610,4609	$c(x) \vee c(x) = c(x)$	[3580:4501,331]
4647,4646	$x \vee x = x$	[4512:4610,4503]
4833,4832	$x \wedge y = c(c(x) \vee c(y))$	[2981:4647,3283,36,4647]
4834	$\$Ans(DM)$	[4832,2]
4845,4844	$(x\|y) = c(x) \vee c(y)$	[2936:4833,4833,4647,466,33,4647,18,4647,4647]
4846	$\$Ans(DEF_SS)$	[4844,5]
4850,4849	$c(x \vee (y \vee x)) \vee z = c(y \vee x) \vee z$	[2652:4647]
4905	$c(c(x \vee c(y)) \vee c(x \vee y)) = x$	[1384:4647,4833,4850,4647]
4907	$\$Ans(CUT_rewritten)$	[4905,4]
4919,4918	$c(x \vee c(y)) \vee c(x \vee y) = c(x)$	[845:4647,4845,4850]
4922	$c(c(x) \vee y) \vee x = x$	[640:4845,33,4647,4845,33,4647,4647,4845,4833,33,4647,33,4647,4845,4919,33,4647]
4931,4930	$c(c(x)) = x$	[32:4647]
4981	$c((x \vee (c(y) \vee z)) \vee c(x \vee (c(y) \vee z))) \vee y = y$	[4115:4833,4931,4833,4931,4931,4845,4931,4556,4845,4931]
5019	$c(x \vee (c(x \vee y) \vee c(y))) \vee x = x \vee y$	[790:4833,4845,4931,4931,4845,4931]
5074	$x \vee (y \vee x) = y \vee x$	[4646 → 927]
5078	$x \vee (c(x \vee y) \vee y) = (x \vee y) \vee c(x \vee y)$	[927 → 4555]
5088	$x \vee (y \vee c(y)) = c(y) \vee (x \vee y)$	[4555 → 927]
5095	$c(x) \vee (y \vee x) = y \vee (x \vee c(x))$	[5088]
5116	$c(x \vee y) \vee c(x) = c(x)$	[4930 → 4922]
5122	$c(x \vee (c(y) \vee z)) \vee y = y$	[927 → 4922]
5151	$c(x \vee c(y)) \vee y = y$	[5074 → 4922]
5157,5156	$c(x \vee y) \vee c(y) = c(y)$	[4930 → 5151]
5160	$c(x \vee c(y)) \vee x = x \vee y$	[5019:5157]

5237	$c(x \vee (y \vee c(z))) \vee z = z$	[5074 → 5122]
5257,5256	$c(x \vee y) \vee x = x \vee c(y)$	[4930 → 5160]
5260,5259	$c(x \vee y) \vee y = y \vee c(x \vee y)$	[5156 → 5160,4931]
5263	$x \vee c(x \vee y) = x \vee c(y)$	[5116 → 5160,4931,5257]
5268,5267	$(x \vee y) \vee c(x \vee y) = x \vee (y \vee c(x \vee y))$	[5078:5260]
5271	$c(x \vee ((c(y) \vee z) \vee c(x \vee (c(y) \vee z)))) \vee y = y$	[4981:5268]
5301	$x \vee c(y \vee c(x)) = x$	[5237 → 5256]
5303	$x \vee c(c(x) \vee y) = x$	[5122 → 5256]
5329	$c(x) \vee c(y \vee x) = c(x)$	[4930 → 5301]
5349,5348	$x \vee (y \vee c(x)) = x \vee c(x)$	[5301 → 5256,4556,4931]
5366	$(x \vee y) \vee y = x \vee y$	[5156 → 5303,4931]
5370	$(x \vee y) \vee x = x \vee y$	[5116 → 5303,4931]
5417,5416	$x \vee (y \vee c(x \vee y)) = (x \vee y) \vee c(y)$	[5366 → 5256,4556,5268]
5431	$c((x \vee (c(y) \vee z)) \vee c(c(y) \vee z)) \vee y = y$	[5271:5417]
5435	$(x \vee y) \vee c(x \vee y) = (x \vee y) \vee c(y)$	[5267:5417]
5499,5498	$c(x) \vee (y \vee x) = x \vee c(x)$	[5329 → 5256,4931,4931]
5506,5505	$x \vee (y \vee c(y)) = y \vee c(y)$	[5095:5499]
5547,5546	$x \vee (y \vee c(x \vee z)) = y \vee (x \vee c(z))$	[5263 → 927]
5549,5548	$(x \vee y) \vee c(y) = y \vee c(y)$	[5416:5547,5349]
5553,5552	$(x \vee y) \vee c(x \vee y) = y \vee c(y)$	[5435:5549]
5557,5556	$c(x \vee c(x)) \vee y = y$	[5431:5549,5553]
5572	$x \vee c(y \vee c(y)) = x$	[5556 → 5370,5557]
5595	$x \vee c(x) = y \vee c(y)$	[5572 → 5263,4931,5506]
5596	$\$Ans(ONE)$	[5595,3]

Proof BA-SS-2

1	$(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C)))$ | $\$Ans(A_SS)$	[]
2	$((A\|(B\|B))\|(A\|B)) \neq A$ | $\$Ans(CUT_SS)$	[]
3	$A \vee B \neq ((A\|A)\|(B\|B))$ | $\$Ans(DEF_J)$	[]
4	$A \wedge B \neq ((A\|B)\|(A\|B))$ | $\$Ans(DEF_M)$	[]
5	$c(A) \neq (A\|A)$ | $\$Ans(DEF_C)$	[]
7	$x \vee (y \vee z) = y \vee (x \vee z)$	[]
9,8	$x \wedge y = c(c(x) \vee c(y))$	[]
11	$(x \vee c(y)) \wedge (x \vee y) = x$	[]
12	$c(c(x \vee c(y)) \vee c(x \vee y)) = x$	[11,9]
14	$(x\|y) = c(x) \vee c(y)$	[]
16,15	$c(x) \vee c(y) = (x\|y)$	[14]
17	$x \vee (x \wedge y) = x$	[]
18	$x \vee c((x\|y)) = x$	[17,9,16]
21,20	$c(c(x)) = x$	[]
22	$c((x \vee c(y)\|x \vee y)) = x$	[12:16]
24	$x \wedge y = c((x\|y))$	[8:16]
34	$x \vee c(y) = (c(x)\|y)$	[20 → 15]
42	$x \vee (y\|z) = c(y) \vee (x \vee c(z))$	[15 → 7]
43	$c(x) \vee (y \vee c(z)) = y \vee (x\|z)$	[42]
84	$c(x) = (x \vee c(y)\|x \vee y)$	[22 → 20]
90	$(x \vee c(y)\|x \vee y) = c(x)$	[84]
94	$x \vee y = (c(x)\|c(y))$	[20 → 34]
96	$(c(x)\|(x\|y)) = x$	[18 → 34]
99	$(c(x)\|c(y)) = x \vee y$	[94]
110,109	$c(x) = (x\|x)$	[96 → 96,21]

111	$\$Ans(DEF_C)$	[109,5]
122,121	$x \vee y = ((x\|x)\|(y\|y))$	[99:110,110]
123	$\$Ans(DEF_J)$	[121,3]
125,124	$((x\|x)\|(x\|y)) = x$	[96:110]
131	$(((x\|x)\|y)\|((x\|x)\|(y\|y))) = (x\|x)$	[90:110,122,125,122,110]
149	$(x\|(((y\|y)\|z)\|((y\|y)\|z))) = ((y\|y)\|((x\|z)\|(x\|z)))$	[43:110,110,122,125,122,125,122]
154	$x \wedge y = ((x\|y)\|(x\|y))$	[24:110]
156	$\$Ans(DEF_M)$	[154,4]
161	$((x\|y)\|(x\|(y\|y))) = x$	[124 $\rightarrow$ 131,125,125]
221	$((x\|(y\|y))\|(x\|y)) = x$	[124 $\rightarrow$ 161]
223	$\$Ans(CUT_SS)$	[221,2]
424	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[124 $\rightarrow$ 149,125,125]
425	$\$Ans(A_SS)$	[424,1]

The following two Mace2 jobs show that the $\mathcal{BA}$ 2-basis {A_SS, CUT_SS} is independent.

Countermodel BA-SS-a

```
f(f(x,f(y,y)),f(x,y)) = x.                          % CUT_SS
f(A,f(f(B,C),f(B,C))) != f(B,f(f(A,C),f(A,C))).  % denial of A_SS
```

C: 0 A: 0 B: 1

f:	0	1
0	0	0
1	1	1

Table 28: BA-SS-a.out

Countermodel BA-SS-b

```
f(x,f(f(y,z),f(y,z))) = f(y,f(f(x,z),f(x,z))).   % A_SS
f(f(A,f(B,B)),f(A,B)) != A.                       % denial of CUT_SS
```

B: 0 A: 1

f:	0	1
0	0	0
1	0	0

Table 29: BA-SS-b.out

2.3 Finding and Proving the Multiequation Bases

See the primary paper [8].

2.4 Are There Simpler Multiequation Bases?

See the primary paper [8].

3 Single Axioms

See the primary paper [8].

3.1 Generating and Filtering Candidates

See the primary paper [8].

3.2 Finite Ortholattices

See the primary paper [8].

3.3 Collecting and Applying Filters

We list here five filter sets. The first four, non-OL.A-4, non-OL.B-9, non-OL.C-23, and non-OL.D-14, are all non-ortholattices, meaning that they can be used to eliminate any $\mathcal{OL}$, $\mathcal{OML}$, $\mathcal{MOL}$, or $\mathcal{BA}$ candidate.

The fifth set, non-MOL-OML, consists of nonmodular orthomodular lattices, which can be used to eliminate $\mathcal{MOL}$ or $\mathcal{BA}$ candidates.

Filter Set non-OL.A-4

f:	0	1	2
0	2	1	0
1	1	1	0
2	0	0	0

Table 30: Structure 1 of non-OL.A-4

f:	0	1	2	3	4	5
0	5	2	4	1	3	0
1	3	3	0	0	3	0
2	1	0	1	1	0	0
3	4	0	4	4	0	0
4	2	2	0	0	2	0
5	0	0	0	0	0	0

Table 31: Structure 2 of non-OL.A-4

f:	0	1	2	3	4	5
0	5	3	1	4	2	0
1	2	3	0	0	2	0
2	4	0	1	4	0	0
3	1	0	1	4	0	0
4	3	3	0	0	2	0
5	0	0	0	0	0	0

Table 32: Structure 3 of non-OL.A-4

f:	0	1	2	3	4	5	6	7
0	7	6	5	4	3	2	1	0
1	6	6	0	0	3	2	0	0
2	5	0	5	0	3	0	1	0
3	4	0	0	4	0	2	1	0
4	3	3	3	0	3	0	0	0
5	2	2	0	2	0	2	0	0
6	1	0	1	1	0	0	1	0
7	0	0	0	0	0	0	0	0

Table 33: Structure 4 of non-OL.A-4

Filter Set non-OL.B-9

f:	0	1	2
0	2	1	0
1	1	1	0
2	0	0	0

Table 34: Structure 1 of non-OL.B-9

f:	0	1	2	3
0	1	0	3	2
1	0	0	0	0
2	3	0	1	0
3	2	0	0	1

Table 35: Structure 2 of non-OL.B-9

f:	0	1	2	3
0	1	1	3	2
1	1	0	0	0
2	3	0	3	0
3	2	0	0	2

Table 36: Structure 3 of non-OL.B-9

f:	0	1	2	3
0	3	2	1	0
1	2	2	0	0
2	1	0	1	1
3	0	0	0	0

Table 37: Structure 4 of non-OL.B-9

f:	0	1	2	3
0	0	2	0	2
1	0	2	0	2
2	1	3	1	3
3	1	3	1	3

Table 38: Structure 5 of non-OL.B-9

f:	0	1	2	3
0	3	2	1	0
1	2	2	0	0
2	1	0	1	0
3	0	0	1	0

Table 39: Structure 6 of non-OL.B-9

f:	0	1	2	3	4
0	0	2	1	4	3
1	2	3	0	1	4
2	1	0	4	3	2
3	4	1	3	2	0
4	3	4	2	0	1

Table 40: Structure 7 of non-OL.B-9

f:	0	1	2	3	4	5
0	5	2	4	1	3	0
1	3	3	0	0	3	0
2	1	0	1	1	0	0
3	4	0	4	4	0	0
4	2	2	0	0	2	0
5	0	0	0	0	0	0

Table 41: Structure 8 of non-OL.B-9

f:	0	1	2	3	4	5
0	5	4	3	2	1	0
1	4	4	3	2	0	0
2	3	3	3	0	1	0
3	2	2	0	2	1	0
4	1	0	1	1	1	0
5	0	0	0	0	0	0

Table 42: Structure 9 of non-OL.B-9

Filter Set non-OL.C-23

f:	0	1	2
0	2	1	0
1	1	1	0
2	0	0	0

Table 43: Structure 1 of non-OL.C-23

f:	0	1	2
0	0	1	2
1	2	0	1
2	1	2	0

Table 44: Structure 2 of non-OL.C-23

f:	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Table 45: Structure 3 of non-OL.C-23

f:	0	1	2
0	1	0	1
1	2	2	1
2	2	0	0

Table 46: Structure 4 of non-OL.C-23

f:	0	1	2
0	1	2	2
1	0	2	0
2	1	1	0

Table 47: Structure 5 of non-OL.C-23

f:	0	1	2	3
0	1	1	3	2
1	1	0	0	0
2	3	0	3	0
3	2	0	0	2

Table 48: Structure 6 of non-OL.C-23

f:	0	1	2	3
0	2	3	0	1
1	2	3	0	1
2	0	0	0	0
3	1	1	1	1

Table 49: Structure 7 of non-OL.C-23

f:	0	1	2	3
0	3	2	1	0
1	2	2	0	0
2	1	0	1	1
3	0	0	0	0

Table 50: Structure 8 of non-OL.C-23

f:	0	1	2	3
0	0	2	0	2
1	0	2	0	2
2	1	3	1	3
3	1	3	1	3

Table 51: Structure 9 of non-OL.C-23

f:	0	1	2	3
0	0	3	1	2
1	2	1	3	0
2	3	0	2	1
3	1	2	0	3

Table 52: Structure 10 of non-OL.C-23

f:	0	1	2	3	4
0	0	2	3	4	1
1	3	1	4	2	0
2	4	0	2	1	3
3	1	4	0	3	2
4	2	3	1	0	4

Table 53: Structure 11 of non-OL.C-23

f:	0	1	2	3	4
0	0	3	4	1	2
1	2	1	0	4	3
2	3	4	2	0	1
3	4	2	1	3	0
4	1	0	3	2	4

Table 54: Structure 12 of non-OL.C-23

f:	0	1	2	3	4
0	0	2	4	1	3
1	1	3	0	2	4
2	2	4	1	3	0
3	3	0	2	4	1
4	4	1	3	0	2

Table 55: Structure 13 of non-OL.C-23

f:	0	1	2	3	4
0	0	2	1	4	3
1	2	3	0	1	4
2	1	0	4	3	2
3	4	1	3	2	0
4	3	4	2	0	1

Table 56: Structure 14 of non-OL.C-23

f:	0	1	2	3	4	5
0	5	3	4	1	2	0
1	3	3	4	0	0	0
2	4	3	4	0	0	0
3	1	0	0	1	2	0
4	2	0	0	1	2	0
5	0	0	0	0	0	0

Table 57: Structure 15 of non-OL.C-23

f:	0	1	2	3	4	5
0	5	3	4	1	2	0
1	3	3	3	0	0	0
2	4	4	4	0	0	0
3	1	0	0	1	1	0
4	2	0	0	2	2	0
5	0	0	0	0	0	0

Table 58: Structure 16 of non-OL.C-23

f:	0	1	2	3	4	5
0	5	4	3	2	1	0
1	4	4	3	2	0	0
2	3	3	3	0	1	0
3	2	2	0	2	1	0
4	1	0	1	1	1	0
5	0	0	0	0	0	0

Table 59: Structure 17 of non-OL.C-23

f:	0	1	2	3	4	5
0	5	2	4	1	3	0
1	3	3	0	0	3	0
2	1	0	1	1	0	0
3	4	0	4	4	0	0
4	2	2	0	0	2	0
5	0	0	0	0	0	0

Table 60: Structure 18 of non-OL.C-23

f:	0	1	2	3	4	5	6
0	0	2	1	4	3	6	5
1	3	5	4	0	2	1	6
2	4	3	6	1	0	5	2
3	5	1	3	2	6	0	4
4	6	4	2	5	1	3	0
5	2	6	0	3	5	4	1
6	1	0	5	6	4	2	3

Table 61: Structure 19 of non-OL.C-23

f:	0	1	2	3	4	5	6
0	0	2	1	4	3	6	5
1	2	1	0	5	6	3	4
2	1	0	2	6	5	4	3
3	4	5	6	3	0	1	2
4	3	6	5	0	4	2	1
5	6	3	4	1	2	5	0
6	5	4	3	2	1	0	6

Table 62: Structure 20 of non-OL.C-23

f:	0	1	2	3	4	5	6	7
0	7	3	6	1	5	4	2	0
1	3	3	3	0	5	4	2	0
2	6	3	6	1	1	1	0	0
3	1	0	1	1	1	1	2	0
4	5	5	1	1	5	0	0	0
5	4	4	1	1	0	4	0	0
6	2	2	0	2	0	0	2	0
7	0	0	0	0	0	0	0	0

Table 63: Structure 21 of non-OL.C-23

f:	0	1	2	3	4	5	6	7	8	9	10	11
0	3	3	5	6	6	9	3	6	3	5	6	9
1	3	4	6	8	6	6	4	6	3	6	8	6
2	5	6	5	6	6	6	5	6	6	5	6	6
3	6	8	6	0	1	0	0	6	1	0	8	6
4	6	6	6	1	1	6	1	6	1	6	6	6
5	9	6	6	0	6	2	2	6	6	0	6	9
6	3	4	5	0	1	2	7	6	10	11	8	9
7	6	6	6	6	6	6	6	6	6	6	6	6
8	3	3	6	1	1	6	10	6	10	6	6	6
9	5	6	5	0	6	0	11	6	6	11	6	6
10	6	8	6	8	6	6	8	6	6	6	8	6
11	9	6	6	6	6	9	9	6	6	6	6	9

Table 64: Structure 22 of non-OL.C-23

f:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	3	3	3	6	6	6	3	6	3	6	6	6	6	6	6	6	3	6
1	3	4	10	8	6	6	4	6	3	8	14	10	10	6	10	14	3	6
2	3	10	5	16	6	6	5	6	3	6	12	10	10	12	10	6	3	16
3	6	8	16	0	1	2	0	6	1	8	1	6	2	6	1	6	2	16
4	6	6	6	1	1	6	1	6	1	6	1	6	6	6	1	6	6	6
5	6	6	6	2	6	2	2	6	6	6	2	6	2	6	6	6	2	6
6	3	4	5	0	1	2	7	6	9	8	11	10	13	12	15	14	17	16
7	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
8	3	3	3	1	1	6	9	6	9	6	1	6	6	6	1	6	3	6
9	6	8	6	8	6	6	8	6	6	8	6	6	6	6	6	6	6	6
10	6	14	12	1	1	2	11	6	1	6	11	6	2	12	1	14	2	6
11	6	10	10	6	6	6	10	6	6	6	6	10	10	6	10	6	6	6
12	6	10	10	2	6	2	13	6	6	6	2	10	13	6	10	6	2	6
13	6	6	12	6	6	6	12	6	6	6	12	6	6	12	6	6	6	6
14	6	10	10	1	1	6	15	6	1	6	1	10	10	6	15	6	6	6
15	6	14	6	6	6	6	14	6	6	6	14	6	6	6	6	14	6	6
16	3	3	3	2	6	2	17	6	3	6	2	6	2	6	6	6	17	6
17	6	6	16	16	6	6	16	6	6	6	6	6	6	6	6	6	6	16

Table 65: Structure 23 of non-OL.C-23

Filter Set non-OL.D-14

f:	0	1	2	3
0	0	1	2	3
1	2	3	0	1
2	3	2	1	0
3	1	0	3	2

Table 66: Structure 1 of non-OL.D-14

f:	0	1	2	3
0	0	1	3	2
1	2	3	1	0
2	0	1	3	2
3	2	3	1	0

Table 67: Structure 2 of non-OL.D-14

f:	0	1	2	3
0	0	0	1	1
1	2	2	3	3
2	1	1	0	0
3	3	3	2	2

Table 68: Structure 3 of non-OL.D-14

f:	0	1	2	3
0	0	2	3	1
1	1	3	2	0
2	2	0	1	3
3	3	1	0	2

Table 69: Structure 4 of non-OL.D-14

f:	0	1	2	3
0	2	2	0	1
1	3	3	0	1
2	0	0	0	1
3	1	1	0	1

Table 70: Structure 5 of non-OL.D-14

f:	0	1	2	3	4
0	0	2	3	4	1
1	4	0	2	1	3
2	1	4	0	3	2
3	2	3	1	0	4
4	3	1	4	2	0

Table 71: Structure 6 of non-OL.D-14

f:	0	1	2	3	4
0	0	2	3	4	1
1	1	3	0	2	4
2	2	1	4	0	3
3	3	4	2	1	0
4	4	0	1	3	2

Table 72: Structure 7 of non-OL.D-14

f:	0	1	2	3	4
0	0	2	3	4	1
1	3	4	2	1	0
2	4	0	1	3	2
3	1	3	0	2	4
4	2	1	4	0	3

Table 73: Structure 8 of non-OL.D-14

f:	0	1	2	3	4
0	0	1	2	3	4
1	2	3	4	0	1
2	4	0	1	2	3
3	1	2	3	4	0
4	3	4	0	1	2

Table 74: Structure 9 of non-OL.D-14

f:	0	1	2	3	4
0	0	1	2	3	4
1	2	3	1	4	0
2	3	0	4	2	1
3	4	2	0	1	3
4	1	4	3	0	2

Table 75: Structure 10 of non-OL.D-14

f:	0	1	2	3	4
0	0	2	3	4	1
1	4	0	1	3	2
2	1	3	0	2	4
3	2	1	4	0	3
4	3	4	2	1	0

Table 76: Structure 11 of non-OL.D-14

f:	0	1	2	3	4
0	0	2	1	4	3
1	3	4	2	0	1
2	4	1	3	2	0
3	2	3	0	1	4
4	1	0	4	3	2

Table 77: Structure 12 of non-OL.D-14

f:	0	1	2	3	4	5	6
0	0	2	3	4	5	6	1
1	6	1	0	5	3	2	4
2	1	5	2	0	6	4	3
3	2	4	6	3	0	1	5
4	3	6	5	1	4	0	2
5	4	3	1	6	2	5	0
6	5	0	4	2	1	3	6

Table 78: Structure 13 of non-OL.D-14

f:	0	1	2	3	4	5	6
0	0	2	1	4	3	6	5
1	3	1	5	0	6	2	4
2	4	6	2	5	0	3	1
3	6	5	4	3	2	1	0
4	5	3	6	1	4	0	2
5	2	4	0	6	1	5	3
6	1	0	3	2	5	4	6

Table 79: Structure 14 of non-OL.D-14

Filter Set non-MOL-OML

f:	0	1	2	3	4	5	6	7	8	9
0	1	1	1	1	1	1	1	1	1	1
1	1	0	5	6	7	2	3	4	9	8
2	1	5	5	1	8	1	1	4	4	8
3	1	6	1	6	1	1	1	1	1	1
4	1	7	8	1	7	2	1	1	2	8
5	1	2	1	1	2	2	1	1	2	1
6	1	3	1	1	1	1	3	1	1	1
7	1	4	4	1	1	1	1	4	4	1
8	1	9	4	1	2	2	1	4	9	1
9	1	8	8	1	8	1	1	1	1	8

Table 80: Structure 1 of non-MOL-OML

f:	0	1	2	3	4	5	6	7	8	9	10	11
0	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	5	6	7	2	3	4	9	8	11	10
2	1	5	5	1	8	1	1	4	4	8	1	1
3	1	6	1	6	1	1	1	1	1	1	1	1
4	1	7	8	1	7	2	1	1	2	8	1	1
5	1	2	1	1	2	2	1	1	2	1	1	1
6	1	3	1	1	1	1	3	1	1	1	1	1
7	1	4	4	1	1	1	1	4	4	1	1	1
8	1	9	4	1	2	2	1	4	9	1	1	1
9	1	8	8	1	8	1	1	1	1	8	1	1
10	1	11	1	1	1	1	1	1	1	1	11	1
11	1	10	1	1	1	1	1	1	1	1	1	10

Table 81: Structure 2 of non-MOL-OML

f:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	5	6	7	2	3	4	9	8	11	10	13	12
2	1	5	5	1	8	1	1	1	10	8	8	10	8	1
3	1	6	1	6	1	1	1	1	1	1	1	1	1	1
4	1	7	8	1	7	1	1	1	12	8	8	1	8	12
5	1	2	1	1	1	2	1	1	2	1	2	1	1	1
6	1	3	1	1	1	1	3	1	1	1	1	1	1	1
7	1	4	1	1	1	1	1	4	4	1	1	1	4	1
8	1	9	10	1	12	2	1	4	9	1	2	10	4	12
9	1	8	8	1	8	1	1	1	1	8	8	1	8	1
10	1	11	8	1	8	2	1	1	2	8	11	1	8	1
11	1	10	10	1	1	1	1	1	10	1	1	10	1	1
12	1	13	8	1	8	1	1	4	4	8	8	1	13	1
13	1	12	1	1	12	1	1	1	12	1	1	1	1	12

Table 82: Structure 3 of non-MOL-OML

f:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	5	6	7	2	3	4	9	8	11	10	13	12
2	1	5	5	1	8	1	1	4	4	8	1	1	1	1
3	1	6	1	6	1	1	1	1	1	1	1	1	1	1
4	1	7	8	1	7	2	1	1	2	8	1	1	1	1
5	1	2	1	1	2	2	1	1	2	1	1	1	1	1
6	1	3	1	1	1	1	3	1	1	1	1	1	1	1
7	1	4	4	1	1	1	1	4	4	1	1	1	1	1
8	1	9	4	1	2	2	1	4	9	1	1	1	1	1
9	1	8	8	1	8	1	1	1	1	8	1	1	1	1
10	1	11	1	1	1	1	1	1	1	1	11	1	1	1
11	1	10	1	1	1	1	1	1	1	1	1	10	1	1
12	1	13	1	1	1	1	1	1	1	1	1	1	13	1
13	1	12	1	1	1	1	1	1	1	1	1	1	1	12

Table 83: Structure 4 of non-MOL-OML

f:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	5	6	7	2	3	4	9	8	11	10	13	12
2	1	5	5	1	8	1	1	4	4	8	1	1	1	1
3	1	6	1	6	1	1	1	1	1	1	12	10	10	12
4	1	7	8	1	7	2	1	1	2	8	1	1	1	1
5	1	2	1	1	2	2	1	1	2	1	1	1	1	1
6	1	3	1	1	1	1	3	1	1	1	3	1	3	1
7	1	4	4	1	1	1	1	4	4	1	1	1	1	1
8	1	9	4	1	2	2	1	4	9	1	1	1	1	1
9	1	8	8	1	8	1	1	1	1	8	1	1	1	1
10	1	11	1	12	1	1	3	1	1	1	11	1	3	12
11	1	10	1	10	1	1	1	1	1	1	1	10	10	1
12	1	13	1	10	1	1	3	1	1	1	3	10	13	1
13	1	12	1	12	1	1	1	1	1	1	12	1	1	12

Table 84: Structure 5 of non-MOL-OML

f:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	5	6	7	2	3	4	9	8	11	10	13	12	15	14
2	1	5	5	1	8	1	1	1	10	8	8	10	8	1	1	1
3	1	6	1	6	1	1	1	1	1	1	1	1	1	1	1	1
4	1	7	8	1	7	1	1	1	12	8	8	1	8	12	1	1
5	1	2	1	1	1	2	1	1	2	1	2	1	1	1	1	1
6	1	3	1	1	1	1	3	1	1	1	1	1	1	1	1	1
7	1	4	1	1	1	1	1	4	4	1	1	1	4	1	1	1
8	1	9	10	1	12	2	1	4	9	1	2	10	4	12	1	1
9	1	8	8	1	8	1	1	1	1	8	8	1	8	1	1	1
10	1	11	8	1	8	2	1	1	2	8	11	1	8	1	1	1
11	1	10	10	1	1	1	1	1	10	1	1	10	1	1	1	1
12	1	13	8	1	8	1	1	4	4	8	8	1	13	1	1	1
13	1	12	1	1	12	1	1	1	12	1	1	1	1	12	1	1
14	1	15	1	1	1	1	1	1	1	1	1	1	1	1	15	1
15	1	14	1	1	1	1	1	1	1	1	1	1	1	1	1	14

Table 85: Structure 6 of non-MOL-OML

f:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	5	6	7	2	3	4	9	8	11	10	13	12	15	14
2	1	5	5	1	8	1	1	1	10	8	8	10	8	1	1	1
3	1	6	1	6	12	1	1	1	12	1	1	1	14	12	12	14
4	1	7	8	12	7	1	1	1	12	8	8	1	8	12	12	1
5	1	2	1	1	1	2	1	1	2	1	2	1	1	1	1	1
6	1	3	1	1	1	1	3	1	1	1	1	1	3	1	3	1
7	1	4	1	1	1	1	1	4	4	1	1	1	4	1	1	1
8	1	9	10	12	12	2	1	4	9	1	2	10	4	12	12	1
9	1	8	8	1	8	1	1	1	1	8	8	1	8	1	1	1
10	1	11	8	1	8	2	1	1	2	8	11	1	8	1	1	1
11	1	10	10	1	1	1	1	1	10	1	1	10	1	1	1	1
12	1	13	8	14	8	1	3	4	4	8	8	1	13	1	3	14
13	1	12	1	12	12	1	1	1	12	1	1	1	1	12	12	1
14	1	15	1	12	12	1	3	1	12	1	1	1	3	12	15	1
15	1	14	1	14	1	1	1	1	1	1	1	1	14	1	1	14

Table 86: Structure 7 of non-MOL-OML

f:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	5	6	7	2	3	4	9	8	11	10	13	12	15	14
2	1	5	5	1	8	1	1	4	4	8	1	1	1	1	1	1
3	1	6	1	6	1	1	1	1	1	1	1	1	1	1	1	1
4	1	7	8	1	7	2	1	1	2	8	1	1	1	1	1	1
5	1	2	1	1	2	2	1	1	2	1	1	1	1	1	1	1
6	1	3	1	1	1	1	3	1	1	1	1	1	1	1	1	1
7	1	4	4	1	1	1	1	4	4	1	1	1	1	1	1	1
8	1	9	4	1	2	2	1	4	9	1	1	1	1	1	1	1
9	1	8	8	1	8	1	1	1	1	8	1	1	1	1	1	1
10	1	11	1	1	1	1	1	1	1	1	11	1	1	1	1	1
11	1	10	1	1	1	1	1	1	1	1	1	10	1	1	1	1
12	1	13	1	1	1	1	1	1	1	1	1	1	13	1	1	1
13	1	12	1	1	1	1	1	1	1	1	1	1	1	12	1	1
14	1	15	1	1	1	1	1	1	1	1	1	1	1	1	15	1
15	1	14	1	1	1	1	1	1	1	1	1	1	1	1	1	14

Table 87: Structure 8 of non-MOL-OML

f:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	5	6	7	2	3	4	9	8	11	10	13	12	15	14
2	1	5	5	1	8	1	1	4	4	8	1	1	1	1	1	1
3	1	6	1	6	1	1	1	1	1	1	1	1	1	1	1	1
4	1	7	8	1	7	2	1	1	2	8	1	1	1	1	1	1
5	1	2	1	1	2	2	1	1	2	1	1	1	1	1	1	1
6	1	3	1	1	1	1	3	1	1	1	1	1	1	1	1	1
7	1	4	4	1	1	1	1	4	4	1	1	1	1	1	1	1
8	1	9	4	1	2	2	1	4	9	1	1	1	1	1	1	1
9	1	8	8	1	8	1	1	1	1	8	1	1	1	1	1	1
10	1	11	1	1	1	1	1	1	1	1	11	1	14	12	12	14
11	1	10	1	1	1	1	1	1	1	1	1	10	10	1	10	1
12	1	13	1	1	1	1	1	1	1	1	14	10	13	1	10	14
13	1	12	1	1	1	1	1	1	1	1	12	1	1	12	12	1
14	1	15	1	1	1	1	1	1	1	1	12	10	10	12	15	1
15	1	14	1	1	1	1	1	1	1	1	14	1	14	1	1	14

Table 88: Structure 9 of non-MOL-OML

3.4 Trying to Prove That Candidates Are Single Axioms

See the primary paper [8].

3.5 Single Axioms for $\mathcal{OL}$, $\mathcal{OML}$, and $\mathcal{MOL}$

The single axioms for the varieties under study are the following.

OL-Sh: $(((y \mid x) \mid (x \mid z)) \mid u) \mid (x \mid ((x \mid ((y \mid y)) \mid y)) \mid z)) = x$
OML-Sh: $(((y \mid x) \mid (x \mid z)) \mid u) \mid (x \mid ((z \mid ((x \mid x) \mid z)) \mid z)) = x$
MOL-Sh: $(y \mid x) \mid (((x \mid x) \mid z) \mid (((((x \mid y) \mid z) \mid z) \mid x) \mid (x \mid u))) = x$

For each, we prove that it is equivalent to the multiequation Sheffer stroke basis for that variety given in Section 2.2.

3.5.1 A Single Axiom for Ortholattices

We list in this subsection three Otter proofs:

1. OL-Sh implies the join/complement basis {AJ, B1, DM, CC, ONE}.
2. OL-Sh implies the Sheffer stroke basis {A_SS, B_SS, ONE_SS}.
3. The Sheffer stroke basis {A_SS, B_SS, ONE_SS} implies OL-Sh.

The first two prove essentially the same theorem—either one is sufficient. For the first proof we have rewritten the goal B1 to eliminate the meet operation; this allows a convenient forward proof.

Proof OL-Sh-jc

1	$A \vee (B \vee C) \neq B \vee (A \vee C) \mid \$Ans(AJ)$	[]
2	$A \vee c(c(A) \vee B) \neq A \mid \$Ans(B1_rewritten)$	[]
3	$A \wedge B \neq c(c(A) \vee c(B)) \mid \$Ans(DM)$	[]
4	$c(c(A)) \neq A \mid \$Ans(CC)$	[]
5	$A \vee c(A) \neq B \vee c(B) \mid \$Ans(ONE)$	[]
6	$(A\|B) \neq c(A) \vee c(B) \mid \$Ans(DEF_SS)$	[]
8	$((((x\|y)\|(y\|z))\|u)\|(y\|((y\|((x\|x)\|x))\|z))) = y$	[]
10	$x \vee y = ((x\|x)\|(y\|y))$	[]
11	$((x\|x)\|(y\|y)) = x \vee y$	[10]
13	$x \wedge y = ((x\|y)\|(x\|y))$	[]
14	$((x\|y)\|(x\|y)) = x \wedge y$	[13]
16	$c(x) = (x\|x)$	[]
18,17	$(x\|x) = c(x)$	[16]
20,19	$c((x\|y)) = x \wedge y$	[14:18]
21	$(c(x)\|c(y)) = x \vee y$	[11:18,18]
23	$((((x\|y)\|(y\|z))\|u)\|(y\|((y\|(c(x)\|x))\|z))) = y$	[8:18]
31,30	$c(c(x)) = x \vee x$	[17 → 21]
35	$c(x) \wedge c(y) = c(x \vee y)$	[21 → 19]
38,37	$c(x \vee x) = c(x) \vee c(x)$	[30 → 30]
41	$(c(x)\|y \vee y) = x \vee c(y)$	[30 → 21]
44,43	$(x \vee x\|c(y)) = c(x) \vee y$	[30 → 21]
45	$(((x \vee y\|(c(y)\|z))\|u)\|(c(y)\|((c(y)\|c(x) \vee x)\|z))) = c(y)$	[21 → 23,31,44]
47	$(((c(x)\|(x\|y))\|z)\|(x\|((x\|(c(x)\|x))\|y))) = x$	[17 → 23]
49	$((((x\|c(y))\|y \vee z)\|u)\|(c(y)\|((c(y)\|(c(x)\|x))\|c(z)))) = c(y)$	[21 → 23]
53	$((x\|y)\|(x\|((x\|((z\|x) \wedge (x\|u)\|((z\|x)\|(x\|u))))\|((x\|(c(z)\|z))\|u)))) = x$	[23 → 23,20]
55	$((x \vee x\|y)\|(x\|((x\|(c(x)\|x))\|x))) = x$	[17 → 23,18,31]
71	$(x \vee x) \wedge c(y) = c(c(x) \vee y)$	[30 → 35]
84,83	$(x \vee x\|y \vee y) = c(x) \vee c(y)$	[30 → 41]
131	$(((x \vee y\|y \vee z)\|u)\|(c(y)\|((c(y)\|c(x) \vee x)\|c(z)))) = c(y)$	[21 → 45]
139	$(((x \vee y\|(c(y)\|(c(y)\|c(x) \vee x)))\|z)\|(c(y)\|c(y) \wedge (c(x) \vee x))) = c(y)$	[17 → 45,20]
151	$(x \vee x) \wedge (y \vee y) = c(c(x) \vee c(y))$	[30 → 71]
259	$(((c(x) \vee y\|y \vee z)\|u)\|(c(y)\|((c(y)\|(x \vee x) \vee c(x))\|c(z)))) = c(y)$	[43 → 49,38,84,31]
263	$((x\|c(y)) \wedge (y \vee z)\|(c(y)\|((c(y)\|(c(x)\|x))\|c(z)))) = c(y)$	[17 → 49,20]
467	$(c(x)\|(x\|((x\|(c(x)\|x))\|y))) = x$	[53 → 47]

469	$(x \vee y \mid (c(y) \mid ((c(y) \mid c(x) \vee x) \mid z))) = c(y)$	[53 → 45]
526,525	$x \vee x = x$	[53 → 467,18,18,31]
531	$(c(x) \wedge (x \mid y) \mid ((c(x) \mid (x \mid y)) \mid x)) = (c(x) \mid (x \mid y))$	[47 → 467,20]
661	$(((c(x) \vee y \mid y \vee z) \mid u) \mid (c(y) \mid ((c(y) \mid x \vee c(x)) \mid c(z)))) = c(y)$	[259:526]
700,699	$x \wedge y = c(c(x) \vee c(y))$	[151:526,526]
701	$\$Ans(DM)$	[699,3]
725,724	$(x \mid y) = c(x) \vee c(y)$	[83:526,526]
726	$\$Ans(DEF_SS)$	[724,6]
727	$c(c(x) \vee c(y)) \vee c(c(x) \vee c(c(c(x) \vee c(x \vee c(x))) \vee c(x))) = x$	[55:526,725,725,31,526,725,725,725,725]
730,729	$c(c(x)) = x$	[30:526]
731	$\$Ans(CC)$	[729,4]
740	$(x \vee c(c(x) \vee c(y))) \vee c(c(x \vee c(c(x) \vee c(y))) \vee c(x)) = x \vee c(c(x) \vee c(y))$	[531:725,700,730,725,725,730,725,725,730,725,725,730]
755,754	$(c(c(x) \vee y) \vee c(y \vee z)) \vee c(y \vee c(c(y \vee c(x \vee c(x))) \vee z)) = c(y)$	[263:725,730,700,725,730,725,730,725,730,725,730,725,730]
762	$c(c(c(x \vee y) \vee c(y \vee c(y \vee c(c(x) \vee x)))) \vee c(z)) \vee c(y \vee (y \vee c(c(x) \vee x))) = c(y)$	[139:725,730,725,730,725,725,700,730,725,730,730,725]
782	$c(c(c(c(x) \vee y) \vee c(y \vee z)) \vee c(u)) \vee c(y \vee c(c(y \vee c(x \vee c(x))) \vee z)) = c(y)$	[661:725,725,725,730,725,730,725,730,725]
794	$c(x \vee y) \vee c(y \vee c(c(y \vee c(c(x) \vee x)) \vee c(z))) = c(y)$	[469:725,730,725,725,730,725]
800	$c(c(c(x \vee y) \vee c(y \vee z)) \vee c(u)) \vee c(y \vee c(c(y \vee c(c(x) \vee x)) \vee z)) = c(y)$	[131:725,725,725,730,725,730,725,730,725]
810	$c(c(x) \vee y) \vee c(c(x) \vee c(c(c(x) \vee c(x \vee c(x))) \vee c(x))) = x$	[729 → 727]
813,812	$c(c(x) \vee c(c(c(x) \vee c(x \vee c(x))) \vee c(x))) = x$	[525 → 727]
816	$c(c(x) \vee y) \vee x = x$	[810:813]
819,818	$c(x \vee y) \vee c(x) = c(x)$	[729 → 816]
822	$(x \vee c(c(x) \vee c(y))) \vee x = x \vee c(c(x) \vee c(y))$	[740:819,730]
831,830	$x \vee (x \vee y) = x \vee y$	[818 → 818,730,730,730]
834	$c(c(c(x \vee y) \vee c(y \vee c(y \vee c(c(x) \vee x)))) \vee c(z)) \vee c(y \vee c(c(x) \vee x)) = c(y)$	[762:831]
871,870	$(x \vee c(c(x) \vee y)) \vee x = x \vee c(c(x) \vee y)$	[729 → 822,730]
872	$((x \vee y) \vee x) \vee (x \vee y) = (x \vee y) \vee x$	[818 → 822,730,819,730]
875,874	$(c(x) \vee c(x \vee y)) \vee c(x) = c(x) \vee c(x \vee y)$	[818 → 872,819,819]
980	$(c(c(x) \vee y) \vee c(y \vee z)) \vee c(y) = c(y)$	[754 → 830,755]
1030	$c((x \vee y) \vee c(z)) \vee c(x \vee c(c(x \vee c(c(x \vee y) \vee (x \vee y))) \vee y)) = c(x)$	[816 → 782,730,730]
1060	$(c(x \vee y) \vee c(y \vee z)) \vee c(y) = c(y)$	[729 → 980]
1072	$x \vee c(c(x) \vee y) = x$	[818 → 980,730,730,871,730]
1074	$\$Ans(B1_rewritten)$	[1072,2]
1075	$c(x) \vee c(x \vee y) = c(x)$	[816 → 980,875]
1093	$(x \vee y) \vee x = x \vee y$	[818 → 1072,730]
1143	$(x \vee c(c(x \vee y) \vee z)) \vee (x \vee y) = x \vee y$	[1075 → 1060,730,730,730]
1151	$(c(x \vee c(y \vee z)) \vee y) \vee (y \vee z) = y \vee z$	[818 → 1060,730,730,730]
1220,1219	$c(x \vee y) \vee c(y) = c(y)$	[818 → 794,730,526]
1243	$c(x \vee c(y)) \vee y = y$	[729 → 1219,730]
1253	$x \vee (y \vee x) = y \vee x$	[1219 → 816,730]
1257	$c(x) \vee c(y \vee x) = c(x)$	[1219 → 1093,1220]
1322,1321	$c(x \vee c(c(x \vee c(c(y) \vee y)) \vee z)) = c(x)$	[1243 → 800]
1335	$c((x \vee y) \vee c(z)) \vee c(x) = c(x)$	[1030:1322]
1359	$c((x \vee y) \vee z) \vee c(x) = c(x)$	[729 → 1335]
1383	$c((c(x) \vee y) \vee z) \vee x = x$	[729 → 1359,730]
1429	$c(x \vee c(c(y) \vee y)) = c(x)$	[1243 → 834]
1439	$c((x \vee c(y)) \vee z) \vee y = y$	[1253 → 1383]
1442,1441	$c(x \vee (c(y) \vee z)) \vee y = y$	[1253 → 1383]
1453	$x \vee (c(y) \vee y) = c(y) \vee y$	[1243 → 1429,730,730,730]

1457	$(c(x) \vee x) \vee y = c(x) \vee x$	[818 → 1429,730,730]
1532,1531	$c(x \vee (y \vee c(z))) \vee z = z$	[1253 → 1439]
1583	$x \vee c(y \vee (c(x) \vee z)) = x$	[1441 → 1093,1442]
1617	$c(x) \vee x = c(y) \vee y$	[1453 → 1457]
1618	$x \vee c(x) = c(y) \vee y$	[729 → 1617]
1619	$c(x) \vee x = y \vee c(y)$	[1618]
1641,1640	$(x \vee y) \vee (y \vee x) = y \vee x$	[1257 → 1151,730]
1656	$x \vee c(x) = y \vee c(y)$	[729 → 1619]
1657	$\$Ans(ONE)$	[1656,5]
1686	$c(x \vee (y \vee z)) \vee c(z) = c(z)$	[729 → 1531]
1710	$x \vee c(y \vee (z \vee c(x))) = x$	[1531 → 1093,1532]
1758	$c(x) \vee c(y \vee (x \vee z)) = c(x)$	[729 → 1583]
1846	$x \vee y = y \vee x$	[1640 → 1093,1641,1641]
1881	$c(x) \vee c(y \vee (z \vee x)) = c(x)$	[729 → 1710]
2012,2011	$(x \vee y) \vee (x \vee (z \vee y)) = x \vee (z \vee y)$	[1686 → 1143,730]
2081	$(x \vee y) \vee (y \vee (x \vee z)) = y \vee (x \vee z)$	[1758 → 1151,730]
2089	$(x \vee y) \vee (y \vee (z \vee x)) = y \vee (z \vee x)$	[1881 → 1151,730]
10276,10275	$(x \vee (y \vee z)) \vee (y \vee x) = x \vee (y \vee z)$	[1846 → 2081]
10439	$(x \vee y) \vee (z \vee y) = z \vee (x \vee y)$	[2011 → 2089,10276,2012]
10467	$x \vee (y \vee z) = (y \vee z) \vee (x \vee z)$	[10439]
10981,10980	$(x \vee y) \vee (z \vee y) = x \vee (z \vee y)$	[1846 → 10439]
11000	$x \vee (y \vee z) = y \vee (x \vee z)$	[10467:10981]
11001	$\$Ans(AJ)$	[11000,1]

Proof OL-Sh

1	$(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C))) \ \mid \ \$Ans(A_SS)$	[]
2	$((A\|A)\|(A\|B)) \neq A \ \mid \ \$Ans(B_SS)$	[]
3	$(A\|(A\|A)) \neq (B\|(B\|B)) \ \mid \ \$Ans(ONE_SS)$	[]
77,76	$((((x\|y)\|(y\|z))\|u)\|(y\|((y\|((x\|x)\|x))\|z))) = y$	[]
79,78	$((x\|y)\|(x\|((x\|((((z\|x)\|(x\|u))\|((z\|x)\|(x\|u)))\|((z\|x)\|(x\|u))))\|((x\|((z\|z)\|z))\|u)))) = x$	[76 → 76]
80	$(x\|((x\|y)\|(((x\|y)\|(((z\|x)\|(z\|x))\|(z\|x)))\|u))) = (x\|y)$	[76 → 76]
82	$((x\|y)\|(((x\|y)\|(y\|z))\|y)) = ((x\|y)\|(y\|z))$	[76 → 80]
84	$(x\|((x\|y)\|(((z\|x)\|(z\|x))\|(z\|x)))) = (x\|y)$	[80 → 80]
87,86	$((((x\|y)\|(y\|z))\|y)\|(y\|y)) = y$	[76 → 82,77]
88	$((x\|x)\|(x\|x)) = x$	[86 → 86]
90	$(x\|((x\|((x\|x)\|y))\|(x\|x))) = (x\|((x\|x)\|y))$	[86 → 82,87,87]
92	$(x\|((x\|y)\|(x\|(x\|x)))) = (x\|y)$	[88 → 84]
94	$((((x\|y)\|(y\|(y\|(y\|y))))\|z)\|(y\|((x\|x)\|x))) = y$	[92 → 76]
96	$(x\|((x\|y)\|(x\|y))) = (x\|y)$	[78 → 80]
99,98	$((x\|y)\|(x\|x)) = x$	[78 → 96,79,79]
100	$((((x\|y)\|(y\|z))\|u)\|(y\|y)) = y$	[76 → 96,77,77]
102	$(x\|((x\|x)\|y)) = (x\|x)$	[90:99]
104	$(((x\|x)\|y)\|x) = (x\|x)$	[98 → 98]
107,106	$((x\|x)\|(x\|y)) = x$	[98 → 102,99]
108	$\$Ans(B_SS)$	[106,2]
110,109	$(((x\|y)\|(y\|z))\|((x\|y)\|(y\|z))) = (((x\|y)\|(y\|z))\|y)$	[76 → 102]
112	$(((x\|y)\|(x\|y))\|x) = (x\|y)$	[98 → 106]
117,116	$(((x\|y)\|(y\|z))\|y) = (y\|((x\|y)\|(y\|z)))$	[76 → 104,110]
119,118	$(((x\|y)\|(y\|z))\|((x\|y)\|(y\|z))) = (y\|((x\|y)\|(y\|z)))$	[109:117]
122	$((((x\|y)\|(x\|z))\|u)\|(x\|x)) = x$	[112 → 100]
124	$(((x\|((x\|y)\|z))\|u)\|((x\|y)\|(x\|y))) = (x\|y)$	[106 → 100]

127,126	$((x\|y)\|(y\|y)) = y$	[98 → 100]
130	$(x\|((y\|x)\|(y\|x))) = (y\|x)$	[126 → 98]
132	$((x\|x)\|(y\|x)) = x$	[126 → 112,127,127]
135,134	$(x\|(y\|(x\|x))) = (x\|x)$	[132 → 132]
137,136	$((x\|y)\|(x\|((z\|z)\|z))) = x$	[94:135,127]
138	$(((x\|((y\|x)\|z))\|u)\|((y\|x)\|(y\|x))) = (y\|x)$	[132 → 100]
140	$((x\|((y\|z)\|(z\|u)))\|(z\|z)) = z$	[130 → 100]
142	$((((x\|y)\|(z\|y))\|u)\|(y\|y)) = y$	[130 → 100]
145,144	$(x\|((x\|y)\|((z\|z)\|z))) = (x\|y)$	[136 → 136]
146	$(x\|x) = (x\|((y\|y)\|y))$	[136 → 132,137]
147	$(x\|((y\|y)\|y)) = (x\|x)$	[146]
150	$(x\|(y\|(y\|y))) = (x\|x)$	[132 → 147]
151	$(x\|x) = (x\|(y\|(y\|y)))$	[150]
152	$(x\|((x\|y)\|(z\|(z\|z)))) = (x\|y)$	[151 → 96]
154	$((x\|(y\|(y\|y)))\|(z\|x)) = x$	[151 → 132]
156	$((x\|((y\|z)\|(y\|u)))\|(y\|y)) = y$	[130 → 122]
159,158	$((x\|(y\|((z\|y)\|u)))\|((z\|y)\|(z\|y))) = (z\|y)$	[154 → 140]
160	$((((x\|y)\|(z\|y))\|u)\|(y\|((v\|v)\|v))) = y$	[146 → 142]
162	$(((x\|(x\|x))\|(y\|(y\|y)))\|z) = (x\|(x\|x))$	[154 → 152]
164	$(x\|(x\|x)) = (y\|(y\|y))$	[126 → 162]
165	$\$Ans(ONE_SS)$	[164,3]
168	$(((x\|y)\|z)\|((x\|((y\|u)\|(y\|v)))\|(x\|((y\|u)\|(y\|v))))) = (x\|((y\|u)\|(y\|v)))$	[156 → 124]
170	$(((x\|y)\|z)\|((y\|x)\|(y\|x))) = (y\|x)$	[136 → 138]
172	$((x\|y)\|(y\|x)) = ((y\|x)\|(y\|x))$	[170 → 104]
173	$((x\|y)\|(((y\|x)\|z)\|((u\|u)\|u))) = ((y\|x)\|z)$	[170 → 136]
176,175	$(x\|((y\|x)\|(x\|y))) = (y\|x)$	[172 → 172,107,119]
177	$(x\|y) = (y\|x)$	[172 → 172,99,119,176]
178	$(((x\|y)\|z)\|((y\|x)\|(x\|y))) = (y\|x)$	[177 → 170]
180	$(((x\|y)\|z)\|((x\|y)\|(y\|x))) = (y\|x)$	[177 → 170]
186	$((x\|(y\|z))\|((((z\|u)\|(z\|v))\|y)\|(((z\|u)\|(z\|v))\|y))) = (((z\|u)\|(z\|v))\|y)$	[122 → 158]
188	$((x\|y)\|z) = ((y\|x)\|z)$	[177 → 173,145]
189	$(x\|(y\|z)) = ((z\|y)\|x)$	[177 → 188]
190	$((x\|y)\|z) = (z\|(y\|x))$	[189]
191	$(x\|((y\|z)\|u)) = (((z\|y)\|u)\|x)$	[189 → 189]
192	$(((x\|y)\|z)\|u) = (u\|((y\|x)\|z))$	[191]
194,193	$((((x\|y)\|z)\|u)\|(((y\|x)\|z)\|((x\|y)\|z))) = (z\|(x\|y))$	[189 → 178]
195	$((x\|y)\|(z\|u)) = ((y\|x)\|(u\|z))$	[189 → 190]
196	$((((x\|y)\|(z\|u))\|v)\|(((y\|x)\|(u\|z))\|((z\|u)\|(x\|y)))) = ((z\|u)\|(x\|y))$	[195 → 180]
198	$((((x\|(y\|((z\|y)\|u)))\|z)\|v)\|((z\|y)\|(z\|y))) = (z\|y)$	[158 → 168,159,159]
200	$((x\|(((y\|((z\|y)\|u))\|v)\|z))\|((z\|y)\|(z\|y))) = (z\|y)$	[192 → 198]
202	$((x\|(y\|z))\|((((z\|u)\|(z\|v))\|y)\|(((z\|v)\|(z\|u))\|y))) = (((z\|v)\|(z\|u))\|y)$	[177 → 186]
204	$(((((x\|y)\|(x\|z))\|u)\|(((x\|z)\|(x\|y))\|u))\|((u\|x)\|v)) = (((x\|z)\|(x\|y))\|u)$	[190 → 202]
206	$(((x\|((y\|x)\|z))\|(x\|u))\|y) = (y\|x)$	[200 → 204]
208	$(((x\|y)\|(x\|z))\|((u\|y)\|(v\|y))) = (((u\|y)\|(v\|y))\|x)$	[160 → 206]
210,209	$(((x\|y)\|(z\|y))\|((u\|v)\|(u\|y))) = (((x\|y)\|(z\|y))\|u)$	[190 → 208]
212,211	$(((x\|y)\|(z\|y))\|((u\|y)\|(u\|v))) = (((x\|y)\|(z\|y))\|u)$	[177 → 208]
217	$(((x\|y)\|(x\|z))\|((u\|y)\|(v\|y))) = (x\|((u\|y)\|(v\|y)))$	[208 → 196,212,210,194]
229	$(((x\|y)\|(x\|y))\|z) = (x\|((z\|y)\|(z\|y)))$	[211 → 217]
230	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[190 → 229]
231	$\$Ans(A_SS)$	[230,1]

Proof OL-Sh-sound

2	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[]
4,3	$((x\|x)\|(x\|y)) = x$	[]
6	$(x\|(x\|x)) = 1$	[]
8	$((((B\|A)\|(A\|C))\|D)\|(A\|((A\|((B\|B)\|B))\|C))) \neq A \mid \$Ans(OL_Sh)$	[]
13	$(x\|((x\|x)\|y)) = (x\|x)$	[3 → 3]
15	$((x\|x), 1) = x$	[6 → 3]
20,19	$(x\|((((x\|y)\|(x\|y))\|y)\|(((x\|y)\|(x\|y))\|y))) = (x\|y)$	[2 → 3]
22,21	$((x\|x)\|x) = 1$	[3 → 6]
23	$(x\|((y\|x)\|(y\|x))) = (y\|x)$	[3 → 2]
25	$((x\|x)\|((y\|(x\|z))\|(y\|(x\|z)))) = (y\|(x\|x))$	[3 → 2,4]
27	$((((B\|A)\|(A\|C))\|D)\|(A\|((A,1)\|C))) \neq A \mid \$Ans(OL_Sh)$	[8:22]
28	$(x,1) = (x\|x)$	[3 → 15]
30,29	$((x\|y)\|(x\|y)) = ((x\|y),1)$	[2 → 15,20]
31	$(x\|x) = (x,1)$	[28]
32	$((x\|x)\|((y\|(x\|z)),1)) = (y\|(x\|x))$	[25:30]
34	$(x\|((y\|x),1)) = (y\|x)$	[23:30]
44	$(x\|((y\|z),1)) = (y\|((x\|z),1))$	[2:30,30]
45	$((x,1)\|x) = 1$	[31 → 21]
48,47	$((x,1),1) = x$	[31 → 15]
50,49	$(x\|((x,1)\|y)) = (x\|x)$	[31 → 13]
55	$((((B\|A)\|(A\|C))\|D)\|(A\|A)) \neq A \mid \$Ans(OL_Sh)$	[27:50]
60	$(x\|y) = (y\|x)$	[45 → 32,30,48,48,30,48]
68,67	$((x\|y)\|(x\|x)) = x$	[31 → 32,48,4]
76,75	$(1\|(x,1)) = x$	[47 → 60]
79	$(x\|x) = (1\|x)$	[28 → 60]
88,87	$((1\|x),1) = x$	[60 → 47]
91	$((x\|x)\|(1\|(y\|(x\|z)))) = (y\|(x\|x))$	[60 → 32]
95,94	$((x\|x)\|(y\|x)) = x$	[60 → 3]
96	$((x\|x)\|((y\|(z\|x)),1)) = (y\|(x\|x))$	[60 → 32]
100,99	$(1\|(1\|x)) = x$	[60 → 75]
113	$(x\|((x\|y),1)) = (y\|x)$	[60 → 34]
115	$((x\|y)\|(x,1)) = x$	[3 → 34,4]
166	$((x\|y)\|(y\|y)) = y$	[60 → 67]
178	$(((x\|y),1)\|((z\|x),1)) = (z\|((x\|y),1))$	[67 → 32,30,30]
180	$((1\|x)\|(y\|x)) = x$	[79 → 94]
184	$((x\|(y\|z))\|(x\|(y\|y))) = ((x\|(y\|z)),1)$	[32 → 94,30,48]
213	$(x\|(y,1)) = ((y\|y)\|x)$	[15 → 44,48]
221	$((x\|x)\|y) = (y\|(x,1))$	[213]
267	$((x\|(y,1))\|y) = (y,1)$	[115 → 166]
269	$((x\|y)\|(1\|y)) = y$	[79 → 166]
340	$((((A\|B)\|(A\|C))\|D)\|(A\|A)) \neq A \mid \$Ans(OL_Sh)$	[60 → 55]
391	$((x\|y)\|(y\|x)) = ((x\|y),1)$	[113 → 180,76]
810,809	$((x\|(y\|y))\|(x\|(y\|z))) = (1\|(x\|(y\|z)))$	[91 → 269,100]
1000	$((x\|((y\|z),1))\|(z\|z)) = z$	[267 → 96,48,95]
2382	$((x\|((y\|z),1))\|(x\|y)) = ((x\|y),1)$	[178 → 267]
2840,2839	$((x\|(y\|y)),1) = (1\|(x\|(y,1)))$	[221 → 391,810]
3192	$((x\|(y\|(z\|u)))\|(1\|(y\|(z,1)))) = (y\|(z\|z))$	[184 → 1000,48,30,2840]
4965	$((D\|((A\|B)\|(A\|C)))\|(A\|A)) \neq A \mid \$Ans(OL_Sh)$	[60 → 340]
20958	$((D\|((A\|B)\|(A\|C)))\|(1\|A)) \neq A \mid \$Ans(OL_Sh)$	[79 → 4965]
38024	$((x\|((y\|z)\|(y\|u)))\|(1\|y)) = y$	[2382 → 3192,88,48,88,68]
38026	$\$Ans(OL_Sh)$	[38024,20958]

3.5.2 A Single Axiom for Orthomodular Lattices

As in the $\mathcal{OL}$ case, we list three Otter proofs:

1. OML-Sh implies the join/complement basis {AJ, B1, DM, OM}.
2. OML-Sh implies the Sheffer stroke basis {A_SS, B_SS, OM_SS}.
3. The Sheffer stroke basis {A_SS, B_SS, OM_SS} implies OML-Sh.

The first two prove essentially the same theorem—either one is sufficient. For the first proof we have rewritten goals B1 and OM to eliminate the meet operation; this allows a convenient forward proof.

Proof OML-Sh-jc

1	$A \vee (B \vee C) \neq B \vee (A \vee C) \mid \$Ans(AJ)$	[]
2	$A \vee c(c(A) \vee B) \neq A \mid \$Ans(B1_rewritten)$	[]
3	$A \wedge B \neq c(c(A) \vee c(B)) \mid \$Ans(DM)$	[]
4	$A \vee c(A \vee c(A \vee B)) \neq A \vee B \mid \$Ans(OM_rewritten)$	[]
5	$(A\|B) \neq c(A) \vee c(B) \mid \$Ans(DEF_SS)$	[]
7	$((((x\|y)\|(y\|z))\|u)\|(y\|((z\|((y\|y)\|z))\|z))) = y$	[]
9	$x \vee y = ((x\|x)\|(y\|y))$	[]
10	$((x\|x)\|(y\|y)) = x \vee y$	[9]
12	$x \wedge y = ((x\|y)\|(x\|y))$	[]
13	$((x\|y)\|(x\|y)) = x \wedge y$	[12]
15	$c(x) = (x\|x)$	[]
17,16	$(x\|x) = c(x)$	[15]
19,18	$c((x\|y)) = x \wedge y$	[13:17]
20	$(c(x)\|c(y)) = x \vee y$	[10:17,17]
22	$((((x\|y)\|(y\|z))\|u)\|(y\|((z\|(c(y)\|z))\|z))) = y$	[7:17]
30,29	$c(c(x)) = x \vee x$	[16 → 20]
34	$c(x) \wedge c(y) = c(x \vee y)$	[20 → 18]
37,36	$c(x \vee x) = c(x) \vee c(x)$	[29 → 29]
40	$(c(x)\|y \vee y) = x \vee c(y)$	[29 → 20]
43,42	$(x \vee x\|c(y)) = c(x) \vee y$	[29 → 20]
46	$(((c(x)\|(x\|y))\|z)\|(x\|((y\|(c(x)\|y))\|y))) = x$	[16 → 22]
48	$((((x\|c(y))\|y \vee z)\|u)\|(c(y)\|((c(z)\|c(y) \vee z)\|c(z)))) = c(y)$	[20 → 22,30,43]
50	$((((x\|y)\|c(y))\|z)\|(y\|((y\|(c(y)\|y))\|y))) = y$	[16 → 22]
52	$((x\|y)\|(x\|((((z\|(c(x)\|z))\|z)\|(c(x)\|((z\|(c(x)\|z))\|z)))\|((z\|(c(x)\|z))\|z)))) = x$	[22 → 22]
54	$((x \vee x\|y)\|(x\|((x\|(c(x)\|x))\|x))) = x$	[16 → 22,17,30]
70	$(x \vee x) \wedge c(y) = c(c(x) \vee y)$	[29 → 34]
83,82	$(x \vee x\|y \vee y) = c(x) \vee c(y)$	[29 → 40]
90	$(((x \vee c(y)\|(y \vee y\|z))\|u)\|(y \vee y\|((z\|(c(y) \vee c(y)\|z))\|z))) = y \vee y$	[40 → 22,37]
136	$(x \vee x) \wedge (y \vee y) = c(c(x) \vee c(y))$	[29 → 70]
158	$(((c(x) \vee c(x)\|c(x) \vee y)\|z)\|(x \vee x\|((c(y)\|(x \vee x) \vee y)\|c(y)))) = x \vee x$	[42 → 46,37,37,43,30]
234	$((x\|c(y)) \wedge (y \vee z)\|(c(y)\|((c(z)\|c(y) \vee z)\|c(z)))) = c(y)$	[16 → 48,19]
306	$(((x \vee c(y)\|c(y) \vee c(y))\|z)\|(y \vee y\|((y \vee y\|(y \vee y) \vee c(y))\|y \vee y))) = y \vee y$	[40 → 50,37,37,83,30]
398	$(c(x)\|(x\|((y\|(c(x)\|y))\|y))) = x$	[52 → 46]
403,402	$((x\|y)\|(y\|((z\|(c(y)\|z))\|z))) = y$	[52 → 22]
451,450	$x \vee x = x$	[398 → 398,403,17,17,30]
456	$(c(x)\|(x\|((c(y)\|x \vee y)\|c(y)))) = x$	[20 → 398]
536	$(((x \vee c(y)\|c(y))\|z)\|(y\|((y\|y \vee c(y))\|y))) = y$	[306:451,451,451,451,451,451]

600	$(((c(x)\|c(x) \vee y)\|z)\|(x\|((c(y)\|x \vee y)\|c(y)))) = x$	[158:451,451,451,451]
603,602	$x \wedge y = c(c(x) \vee c(y))$	[136:451,451]
604	$\$Ans(DM)$	[602,3]
623	$(((x \vee c(y)\|(y\|z))\|u)\|(y\|((z\|(c(y)\|z))\|z))) = y$	[90:451,451,451,451]
632,631	$(x\|y) = c(x) \vee c(y)$	[82:451,451]
633	$\$Ans(DEF_SS)$	[631,5]
638	$c(c(x) \vee c(y)) \vee c(c(x) \vee c(c(c(x) \vee c(x \vee c(x))) \vee c(x))) = x$	[54:451,632,632,30,451,632,632,632,632]
643,642	$c(c(x)) = x$	[29:451]
654	$(c(c(x) \vee y) \vee c(y \vee z)) \vee c(y \vee c(c(z \vee c(c(y) \vee z)) \vee z)) = c(y)$	[234:632,643,603,632,643,632,643,632,643,632,643]
666	$c(c(c(x \vee c(y)) \vee c(c(y) \vee c(z))) \vee c(u)) \vee c(c(y) \vee c(c(c(z) \vee c(y \vee c(z))) \vee c(z))) = y$	[623:632,632,632,632,643,632,632,632,632]
672	$c(c(x \vee c(c(x) \vee y)) \vee c(z)) \vee c(c(x) \vee c(c(y \vee c(x \vee y)) \vee y)) = x$	[600:632,643,632,632,643,632,643,632,632]
674	$c(c(c(x \vee c(y)) \vee y) \vee c(z)) \vee c(c(y) \vee c(c(c(y) \vee c(y \vee c(y))) \vee c(y))) = y$	[536:632,643,632,632,632,632,632]
682	$x \vee c(c(x) \vee c(c(y \vee c(x \vee y)) \vee y)) = x$	[456:632,643,632,643,632,632,643]
706	$c(c(x) \vee y) \vee c(c(x) \vee c(c(c(x) \vee c(x \vee c(x))) \vee c(x))) = x$	[642 → 638]
709,708	$c(c(x) \vee c(c(c(x) \vee c(x \vee c(x))) \vee c(x))) = x$	[450 → 638]
712	$c(c(x) \vee y) \vee x = x$	[706:709]
714	$c(c(c(x \vee c(y)) \vee y) \vee c(z)) \vee y = y$	[674:709]
719,718	$c(x \vee y) \vee c(x) = c(x)$	[642 → 712]
724	$x \vee (x \vee y) = x \vee y$	[718 → 718,643,643,643]
728	$x \vee c(c(x) \vee c(c((x \vee y) \vee c(x \vee y)) \vee (x \vee y))) = x$	[724 → 682]
738	$(x \vee c(c(x) \vee y)) \vee c(c(x) \vee c(c(y \vee c(x \vee y)) \vee y)) = x$	[718 → 654,643,643,643]
794	$c((c(x) \vee c(y)) \vee c(z)) \vee c(c(x) \vee c(c(c(y) \vee c(x \vee c(y))) \vee c(y))) = x$	[718 → 666,643]
800	$(c(x \vee c(y)) \vee c(c(y) \vee c(z))) \vee c(c(y) \vee c(c(c(z) \vee c(y \vee c(z))) \vee c(z))) = y$	[682 → 666,643]
809,808	$c(x \vee c(y)) \vee y = y$	[718 → 714,643]
818	$x \vee c(c(x) \vee c(x \vee y)) = x$	[728:809]
822	$c(x \vee y) \vee c(y) = c(y)$	[642 → 808]
839,838	$x \vee (y \vee x) = y \vee x$	[822 → 712,643]
877,876	$c(c(x) \vee c(c(y \vee c(x \vee y)) \vee y)) = x$	[808 → 672]
892	$(c(x \vee c(y)) \vee c(c(y) \vee c(z))) \vee y = y$	[800:877]
896	$c((c(x) \vee c(y)) \vee c(z)) \vee x = x$	[794:877]
909,908	$(x \vee c(c(x) \vee y)) \vee x = x$	[738:877]
913,912	$(x \vee c(y \vee c(x))) \vee x = x$	[838 → 908]
919,918	$c(x) \vee c(x \vee c(c(x) \vee y)) = c(x \vee c(c(x) \vee y))$	[908 → 718]
925,924	$c(x \vee c(c(x) \vee y)) = c(x)$	[642 → 818,919]
926	$x \vee c(y \vee c(x)) = x$	[912 → 818,719,643,913]
928	$x \vee c(c(x) \vee y) = x$	[908 → 818,925,451,643,909]
930	$\$Ans(B1_rewritten)$	[928,2]
937	$c(x) \vee c(y \vee x) = c(x)$	[642 → 926]
953	$c(x) \vee c(x \vee y) = c(x)$	[642 → 928]
955	$(x \vee y) \vee y = x \vee y$	[822 → 928,643]
958,957	$(x \vee y) \vee x = x \vee y$	[718 → 928,643]
1017	$c((c(x) \vee y) \vee c(z)) \vee x = x$	[642 → 896]
1035	$c((c(x) \vee y) \vee z) \vee x = x$	[642 → 1017]
1037	$c((x \vee y) \vee z) \vee c(x) = c(x)$	[642 → 1035]
1041	$c((x \vee c(y)) \vee z) \vee y = y$	[838 → 1035]
1051	$c(x \vee (c(y) \vee z)) \vee y = y$	[838 → 1035]
1063	$c(c(x \vee y) \vee c(c(y \vee c(x \vee y)) \vee y)) = x \vee y$	[955 → 876]
1085	$c(x \vee (y \vee c(z))) \vee z = z$	[838 → 1041]
1095	$c(x \vee (y \vee z)) \vee c(y) = c(y)$	[642 → 1051]

1159	$(c(x \vee c(y \vee (z \vee u))) \vee u) \vee (y \vee (z \vee u)) = y \vee (z \vee u)$	[1085 → 892,643,643,643,643]
1161	$(c(x \vee c(y \vee (z \vee u))) \vee z) \vee (y \vee (z \vee u)) = y \vee (z \vee u)$	[1051 → 892,643,643,643,643]
1163	$(c(x \vee c((y \vee z) \vee u)) \vee z) \vee ((y \vee z) \vee u) = (y \vee z) \vee u$	[1041 → 892,643,643,643,643]
1165	$(c(x \vee c((y \vee z) \vee u)) \vee y) \vee ((y \vee z) \vee u) = (y \vee z) \vee u$	[1035 → 892,643,643,643,643]
1168,1167	$(c(x \vee c(y \vee z)) \vee z) \vee (y \vee z) = y \vee z$	[822 → 892,643]
1169	$(c(x \vee c(y \vee z)) \vee y) \vee (y \vee z) = y \vee z$	[718 → 892,643]
1200,1199	$((x \vee y) \vee z) \vee x = (x \vee y) \vee z$	[1037 → 928,643]
1258,1257	$x \vee (y \vee (x \vee z)) = y \vee (x \vee z)$	[1095 → 712,643]
2405	$c(x \vee c(y \vee x)) \vee x = y \vee x$	[1063 → 937,643,1168,643]
2511	$c(x \vee c(x \vee y)) \vee x = x \vee y$	[957 → 2405,958]
4760	$(x \vee y) \vee (x \vee (z \vee y)) = x \vee (z \vee y)$	[953 → 1159,643]
5313,5312	$(x \vee y) \vee (x \vee (y \vee z)) = x \vee (y \vee z)$	[953 → 1161,643]
5739,5738	$(x \vee y) \vee ((z \vee y) \vee x) = (z \vee y) \vee x$	[937 → 1163,643]
6102	$(x \vee y) \vee ((y \vee z) \vee x) = (y \vee z) \vee x$	[937 → 1165,643]
7373,7372	$(x \vee y) \vee (y \vee x) = y \vee x$	[937 → 1169,643]
7594	$x \vee y = y \vee x$	[7372 → 957,7373,7373]
7609	$x \vee c(x \vee c(x \vee y)) = x \vee y$	[2511 → 7594]
7611	$\$Ans(OM_rewritten)$	[7609,4]
17813	$(x \vee (y \vee z)) \vee (x \vee z) = x \vee (y \vee z)$	[7594 → 4760]
18921	$((x \vee y) \vee (y \vee z)) \vee (x \vee (y \vee z)) = x \vee (y \vee z)$	[5312 → 4760,5313]
19351	$((x \vee y) \vee (z \vee y)) \vee ((z \vee y) \vee x) = (z \vee y) \vee x$	[5738 → 5312,5739]
19774,19773	$((x \vee y) \vee z) \vee (z \vee x) = (x \vee y) \vee z$	[7594 → 6102]
19779	$(x \vee y) \vee (z \vee y) = (z \vee y) \vee x$	[19351:19774]
20260	$(x \vee y) \vee (z \vee x) = (z \vee x) \vee y$	[7594 → 19779]
20410,20409	$(x \vee y) \vee (z \vee y) = (x \vee y) \vee z$	[7594 → 19779]
20422	$(x \vee y) \vee z = (y \vee z) \vee (x \vee y)$	[20260]
20472,20471	$(x \vee y) \vee (y \vee z) = x \vee (y \vee z)$	[18921:20410,1200]
20614,20613	$(x \vee y) \vee (z \vee x) = (x \vee y) \vee z$	[19773:20472]
20648	$(x \vee y) \vee z = (y \vee z) \vee x$	[20422:20614]
21300,21299	$(x \vee y) \vee z = x \vee (y \vee z)$	[20648 → 724,20614,20472]
22049	$x \vee (y \vee z) = y \vee (x \vee z)$	[17813:21300,21300,839,1258]
22050	$\$Ans(AJ)$	[22049,1]

Proof OML-Sh

1	$(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C))) \ \mid \ \$Ans(A_SS)$	[]
2	$((A\|A)\|(A\|B)) \neq A \ \mid \ \$Ans(B_SS)$	[]
3	$(A\|(A\|(A\|B))) \neq (A\|B) \ \mid \ \$Ans(OM_SS)$	[]
77,76	$((((x\|y)\|(y\|z))\|u)\|(y\|((z\|((y\|y)\|z))\|z))) = y$	[]
78	$(x\|((x\|y)\|((z\|(((x\|y)\|(x\|y))\|z))\|z))) = (x\|y)$	[76 → 76]
80	$((((x\|y)\|(y\|z))\|u)\|(y\|((v\|((y\|y)\|v))\|v))) = y$	[76 → 78,77,77,77]
83,82	$((x\|y)\|(x\|((z\|((x\|x)\|z))\|z))) = x$	[80 → 80]
85,84	$((x\|y)\|(y\|((z\|((y\|y)\|z))\|z))) = y$	[82 → 80]
87,86	$((x\|y)\|(y\|y)) = y$	[84 → 84,85]
88	$((x\|y)\|(y\|((y\|y)\|z))) = y$	[78 → 84,85]
91,90	$((x\|y)\|(x\|x)) = x$	[84 → 82,85]
92	$((((x\|y)\|(y\|z))\|u)\|(y\|y)) = y$	[84 → 80,85]
94	$(x\|((x\|y)\|(x\|y))) = (x\|y)$	[84 → 78,85]
96	$(x\|((y\|x)\|((z\|(((y\|x)\|(y\|x))\|z))\|z))) = (y\|x)$	[84 → 82]
99,98	$(x\|((y\|((x\|x)\|y))\|y)) = (x\|x)$	[84 → 86,83]
100	$(x\|((y\|x)\|(y\|x))) = (y\|x)$	[96:99]
102	$(((x\|x)\|y)\|x) = (x\|x)$	[90 → 90]

105,104	$((x\|((x\|x)\|y))\|(x\|((x\|x)\|y))) = x$	[102 → 88]
106	$(x\|((x\|x)\|y)) = (x\|x)$	[88 → 86,105]
109,108	$((x\|x)\|(x\|y)) = x$	[90 → 106,91]
110	$\$Ans(B_SS)$	[108,2]
112,111	$(((x\|y)\|(x\|y))\|x) = (x\|y)$	[90 → 108]
113	$((x\|x)\|(y\|x)) = x$	[100 → 108]
115	$(((x\|((y\|x)\|z))\|u)\|((y\|x)\|(y\|x))) = (y\|x)$	[113 → 92]
117	$((((x\|y)\|(x\|z))\|u)\|(x\|x)) = x$	[111 → 92]
119	$(((x\|((x\|y)\|z))\|u)\|((x\|y)\|(x\|y))) = (x\|y)$	[108 → 92]
121	$((((x\|y)\|(z\|y))\|u)\|(y\|y)) = y$	[100 → 92]
123	$((x\|((y\|z)\|(z\|u)))\|(z\|z)) = z$	[100 → 92]
126,125	$(((x\|y)\|(y\|z))\|((x\|y)\|(y\|z))) = (y\|((x\|y)\|(y\|z)))$	[92 → 102]
127	$(x\|(((y\|x)\|x)\|(y\|x))) = (x\|x)$	[113 → 98]
129	$((x\|((y\|z)\|(y\|u)))\|(y\|y)) = y$	[100 → 117]
132,131	$((x\|((y\|z)\|(u\|z)))\|(z\|z)) = z$	[100 → 121]
134,133	$((x\|(y\|((z\|y)\|u)))\|((z\|y)\|(z\|y))) = (z\|y)$	[113 → 123]
135	$(((x\|y)\|y)\|(x\|y)) = y$	[127 → 113,132]
137	$(((x\|y)\|x)\|(x\|y)) = x$	[111 → 135,112]
139	$((x\|(x\|y))\|x) = (x\|y)$	[108 → 135,109]
141	$((x\|y)\|(x\|(x\|y))) = x$	[139 → 137]
143	$(((x\|y)\|z)\|((y\|x)\|(y\|x))) = (y\|x)$	[141 → 115]
145	$((x\|y)\|(y\|x)) = ((y\|x)\|(y\|x))$	[143 → 102]
147,146	$(x\|((y\|x)\|(x\|y))) = (y\|x)$	[145 → 145,109,126]
148	$(x\|y) = (y\|x)$	[145 → 145,91,126,147]
150,149	$((((x\|y)\|(y\|x))\|z)\|(x\|y)) = ((y\|x)\|(x\|y))$	[145 → 143,87]
151	$(x\|(x\|(x\|y))) = (x\|y)$	[139 → 148]
153	$\$Ans(OM_SS)$	[151,3]
154	$((x\|(y\|x))\|(y\|x)) = x$	[148 → 135]
157,156	$(((x\|y)\|z)\|((y\|x)\|(x\|y))) = (y\|x)$	[148 → 143]
158	$(((x\|y)\|z)\|((x\|y)\|(y\|x))) = (y\|x)$	[148 → 143]
161,160	$((x\|(x\|y))\|(y\|x)) = x$	[148 → 154]
163,162	$(((x\|y)\|(y\|x))\|((y\|x)\|z)) = (y\|x)$	[156 → 156,157,150,157]
164	$(((x\|y)\|z)\|((x\|((y\|u)\|(y\|v)))\|(x\|((y\|u)\|(y\|v))))) = (x\|((y\|u)\|(y\|v)))$	[129 → 119]
166	$((x\|(y\|z))\|((((z\|u)\|(z\|v))\|y)\|(((z\|u)\|(z\|v))\|y))) = (((z\|u)\|(z\|v))\|y)$	[117 → 133]
168	$((x\|y)\|(y\|((x\|y)\|(y\|z)))) = ((x\|y)\|(y\|z))$	[125 → 94]
171,170	$((x\|y)\|(y\|((x\|y)\|(z\|y)))) = ((x\|y)\|(y\|z))$	[148 → 168]
172	$((x\|y)\|(y\|z)) = ((x\|y)\|(z\|y))$	[146 → 168,171,147]
173	$(x\|((y\|x)\|z)) = (x\|(z\|(y\|x)))$	[160 → 172,161]
175	$(x\|(y\|z)) = (x\|(z\|y))$	[156 → 173,163]
176	$((x\|y)\|z) = (z\|(y\|x))$	[148 → 175]
177	$(x\|(y\|z)) = ((z\|y)\|x)$	[176]
178	$(x\|((y\|z)\|u)) = (((z\|y)\|u)\|x)$	[177 → 177]
179	$((x\|y)\|(z\|u)) = ((y\|x)\|(u\|z))$	[176 → 177]
181	$(((x\|y)\|z)\|u) = (u\|((y\|x)\|z))$	[178]
183,182	$((((x\|y)\|z)\|u)\|(((y\|x)\|z)\|((x\|y)\|z))) = (z\|(x\|y))$	[177 → 156]
184	$((((x\|y)\|(z\|u))\|v)\|(((y\|x)\|(u\|z))\|((z\|u)\|(x\|y)))) = ((z\|u)\|(x\|y))$	[179 → 158]
186	$((((x\|(y\|((z\|y)\|u)))\|z)\|v)\|((z\|y)\|(z\|y))) = (z\|y)$	[133 → 164,134,134]
188	$((x\|(((y\|((z\|y)\|u))\|v)\|z))\|((z\|y)\|(z\|y))) = (z\|y)$	[181 → 186]
190	$((x\|(y\|z))\|((((z\|u)\|(z\|v))\|y)\|(((z\|v)\|(z\|u))\|y))) = (((z\|v)\|(z\|u))\|y)$	[148 → 166]
192	$(((((x\|y)\|(x\|z))\|u)\|(((x\|z)\|(x\|y))\|u))\|((u\|x)\|v)) = (((x\|z)\|(x\|y))\|u)$	[176 → 190]
194	$(((x\|((y\|x)\|z))\|(x\|u))\|y) = (y\|x)$	[188 → 192]
196	$(((x\|y)\|(x\|z))\|((u\|y)\|(v\|y))) = (((u\|y)\|(v\|y))\|x)$	[121 → 194]
198,197	$(((x\|y)\|(z\|y))\|((u\|v)\|(u\|y))) = (((x\|y)\|(z\|y))\|u)$	[176 → 196]
200,199	$(((x\|y)\|(z\|y))\|((u\|y)\|(u\|v))) = (((x\|y)\|(z\|y))\|u)$	[148 → 196]

205	$(((x\|y)\|(x\|z))\|((u\|y)\|(v\|y))) = (x\|((u\|y)\|(v\|y)))$	[196 → 184,200,198,183]
215	$(((x\|y)\|(x\|y))\|z) = (x\|((z\|y)\|(z\|y)))$	[199 → 205]
216	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[176 → 215]
217	$\$Ans(A_SS)$	[216,1]

Proof OML-Sh-sound

2	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[]
4,3	$((x\|x)\|(x\|y)) = x$	[]
5	$(x\|(x\|(x\|y))) = (x\|y)$	[]
7	$((((B\|A)\|(A\|C))\|D)\|(A\|((C\|((A\|A)\|C))\|C))) \neq A \mid \$Ans(OML_Sh)$	[]
9,8	$(x\|((x\|x)\|y)) = (x\|x)$	[3 → 3]
13,12	$(x\|((y\|x)\|(y\|x))) = (y\|x)$	[3 → 2]
14	$((x\|x)\|((y\|(x\|z))\|(y\|(x\|z)))) = (y\|(x\|x))$	[3 → 2,4]
31,30	$((x\|y)\|(x\|x)) = x$	[3 → 12,4,4]
33,32	$(x\|(y\|(x\|x))) = (x\|x)$	[12 → 8]
34	$(x\|(x\|(y\|x))) = (y\|x)$	[12 → 5,13]
45,44	$(x\|((((x\|y)\|z)\|y)\|(((x\|y)\|z)\|y))) = (x\|y)$	[2 → 30]
46	$((x\|y)\|((x\|y)\|x)) = x$	[30 → 5,31]
48	$(((x\|y)\|(x\|y))\|x) = (x\|y)$	[30 → 3]
50	$((x\|y)\|((z\|(x\|x))\|(z\|(x\|x)))) = (z\|(x\|x))$	[30 → 2,31]
99	$((x\|x)\|y) = (y\|(x\|x))$	[32 → 14,33,4]
103	$((x\|(y\|z))\|(x\|(y\|y))) = ((x\|(y\|z))\|(x\|(y\|z)))$	[14 → 32]
117	$(x\|y) = (y\|x)$	[99 → 99,4,4]
127,126	$(((x\|y)\|x)\|(x\|y)) = x$	[46 → 117]
129,128	$((x\|(y\|x))\|x) = (y\|x)$	[34 → 117]
133	$((((B\|A)\|(A\|C))\|D)\|(A\|A)) \neq A \mid \$Ans(OML_Sh)$	[7:129,9]
146,145	$(x\|((y\|x)\|(x\|y))) = (y\|x)$	[117 → 12]
197	$(((x\|y)\|x)\|x) = (x\|y)$	[126 → 46]
199	$(((x\|y)\|x)\|((z\|(x\|y))\|(z\|(x\|y)))) = (z\|(x\|x))$	[126 → 2,127]
338,337	$(((x\|y)\|(y\|x))\|y) = (y\|x)$	[117 → 48]
342,341	$(((x\|y)\|(y\|x))\|((x\|y)\|(y\|x))) = (x\|y)$	[145 → 145,338,146]
366,365	$((x\|y)\|((z\|(y\|x))\|(z\|(y\|x)))) = (z\|(x\|y))$	[337 → 14,342,338,342]
371	$(x\|(y\|z)) = (x\|(z\|y))$	[337 → 50,342,342,366,342]
376	$(((((x\|y)\|z)\|y)\|u)\|((x\|y)\|(x\|y))) = (x\|y)$	[44 → 50,45,45]
420	$((x\|y)\|z) = (z\|(y\|x))$	[117 → 371]
430	$(x\|(y\|z)) = ((z\|y)\|x)$	[420]
596	$((x\|y)\|(z\|u)) = ((y\|x)\|(u\|z))$	[420 → 430]
3288	$((((A\|B)\|(C\|A))\|D)\|(A\|A)) \neq A \mid \$Ans(OML_Sh)$	[596 → 133]
8007	$(((x\|(y\|y))\|x)\|(x\|(y\|z))) = x$	[103 → 199,4,31]
8087	$(((x\|y)\|y)\|(y\|((x\|y)\|z))) = y$	[12 → 8007]
8651	$((x\|y)\|(x\|(((x\|y)\|x)\|z))) = x$	[197 → 8087]
12837,12836	$((x\|(((x\|y)\|x)\|z))\|(y\|x)) = x$	[420 → 8651]
15661	$((((x\|y)\|(z\|x))\|u)\|(x\|x)) = x$	[12836 → 376,12837,12837,12837]
15663	$\$Ans(OML_Sh)$	[15661,3288]

3.5.3 A Single Axiom for Modular Ortholattices

The proofs given in this subsection are all in terms of the Sheffer stroke. We have simplified the proof of

MOL-Sh ⇒ {A_SS, B_SS, ONE_SS, MOD_SS}

by presenting it in two stages. First we prove $x|(x|x) = y|(y|y)$, which allows us to introduce a constant we call "1" with the property $x|(x|x) = 1$. We use this in the second stage. The third proof is

$\{$A_SS, B_SS, ONE_SS, MOD_SS$\} \Rightarrow$ MOL-Sh

Proof MOL-Sh-part1

1	$(A\|(A\|A)) \neq ((B\|B)\|B) \ \mid \ \$Ans(ONE_SS)$	[]
4,3	$((x\|y)\|(((y\|y)\|z)\|(((((y\|x)\|z)\|z)\|y)\|(y\|u)))) = y$	[]
5	$((x\|(y\|z))\|((((y\|z)\|(y\|z))\|u)\|((((((y\|z)\|x)\|u)\|u)\|(y\|z))\|z))) = (y\|z)$	[3 → 3]
7	$((x\|y)\|(((y\|y)\|z)\|(((((y\|x)\|z)\|z)\|y)\|(((y\|y)\|u)\|(((((y\|v)\|u)\|u)\|y)\|(y\|w))))))) = y$	[3 → 5,4,4,4,4,4]
10,9	$((x\|y)\|(((y\|y)\|z)\|y)) = y$	[3 → 7]
11	$(x\|((x\|y)\|(((x\|x)\|z)\|x))) = (((x\|x)\|z)\|x)$	[9 → 9,10]
16,15	$((x\|y)\|(y\|y)) = y$	[9 → 9]
18,17	$((x\|(((y\|y)\|z)\|y))\|y) = (((y\|y)\|z)\|y)$	[9 → 9]
19	$((x\|(y\|z))\|((((y\|z)\|(y\|z))\|(((x\|x)\|u)\|x))\|(((x\|(((x\|x)\|u)\|x))\|(y\|z))\|z))) = (y\|z)$	[9 → 5]
27	$((x\|y)\|(((y\|y)\|(((x\|x)\|z)\|x))\|(((x\|(((x\|x)\|z)\|x))\|y)\|(y\|u)))) = y$	[9 → 3]
33	$((x\|y)\|(y\|((((y\|y)\|z)\|y)\|(y\|u)))) = y$	[9 → 3,18]
36,35	$(((x\|x)\|y)\|x) = (x\|x)$	[11 → 11,10]
45	$((x\|y)\|(y\|((y\|y)\|(y\|z)))) = y$	[33:36]
51	$((x\|y)\|(((y\|y)\|(x\|x))\|(((x\|(x\|x))\|y)\|(y\|z)))) = y$	[27:36,36]
59	$((x\|(y\|z))\|((((y\|z)\|(y\|z))\|(x\|x))\|(((x\|(x\|x))\|(y\|z))\|z))) = (y\|z)$	[19:36,36]
62,61	$((x\|(y\|y))\|y) = (y\|y)$	[17:36,36]
76,75	$((x\|y)\|(x\|x)) = x$	[15 → 35,16]
83	$((x\|x)\|(((x\|x)\|x)\|(((x\|x)\|x)\|(x\|y)))) = x$	[35 → 3]
88,87	$(x\|((x\|y)\|(x\|y))) = (x\|y)$	[75 → 75]
90,89	$(x\|((y\|x)\|(y\|x))) = (y\|x)$	[15 → 75]
95	$((x\|(y\|z))\|((((y\|z)\|(y\|z))\|u)\|((((((y\|z)\|x)\|u)\|u)\|(y\|z))\|y))) = (y\|z)$	[75 → 3]
97	$((x\|y)\|(((y\|y)\|((y\|x)\|(y\|x)))\|(((y\|x)\|y)\|(y\|z)))) = y$	[75 → 3]
108,107	$((x\|((x\|x)\|(x\|y)))\|(x\|((x\|x)\|(x\|y)))) = x$	[35 → 45]
125	$(x\|((x\|x)\|(x\|y))) = (x\|x)$	[45 → 15,108]
129	$(((x\|x)\|(y\|z))\|((((y\|z)\|(y\|z))\|x)\|((((x\|x)\|x)\|(y\|z))\|z))) = (y\|z)$	[61 → 5]
169	$((x\|y)\|(((y\|y)\|z)\|(((((y\|x)\|z)\|z)\|y)\|(u\|y)))) = y$	[89 → 3]
179	$((x\|x)\|((y\|(x\|x))\|((((y\|(x\|x))\|((y\|(x\|x))\|(y\|(x\|x))))\|x)\|(x\|z)))) = x$	[61 → 51,90]
181	$((x\|x)\|(((x\|x)\|y)\|((((x\|x)\|y)\|(((x\|x)\|y)\|((x\|x)\|y)))\|x)\|(x\|z)))) = x$	[35 → 51,88]
193	$((x\|(y\|z))\|((((y\|z)\|(y\|z))\|(x\|x))\|(((x\|(x\|x))\|(y\|z))\|y))) = (y\|z)$	[75 → 51]
198,197	$((x\|x)\|(x\|((x\|x)\|y))) = x$	[75 → 125,76]
199	$(x\|((x\|x)\|(y\|x))) = (x\|x)$	[89 → 125]
201	$((x\|y)\|(((x\|y)\|(x\|y))\|x)) = ((x\|y)\|(x\|y))$	[75 → 125]
217	$((x\|x)\|(x\|(y\|(x\|x)))) = x$	[89 → 197]
382,381	$((x\|x)\|(((x\|x)\|x)\|(((x\|x)\|x)\|(y\|x)))) = x$	[89 → 83]
449	$((x\|(y\|z))\|((((y\|z)\|(y\|z))\|(((y\|z)\|x)\|((y\|z)\|x)))\|((((y\|z)\|x)\|(y\|z))\|y))) = (y\|z)$	[75 → 95]
482,481	$(((x\|x)\|x)\|(x\|((x\|x)\|y))) = x$	[97 → 59,76,90,16,76,90,16,76,90,16,16,198,62,76,76,90,16]
488,487	$((x\|(x\|x))\|((x\|x)\|(x\|y))) = (x\|x)$	[75 → 481,76]
490,489	$(((x\|x)\|x)\|(x\|(y\|(x\|x)))) = x$	[89 → 481]
504,503	$((x\|(x\|x))\|((x\|x)\|(y\|x))) = (x\|x)$	[75 → 489,76]
516,515	$(((x\|y)\|((x\|y)\|(x\|y)))\|(((x\|y)\|(x\|y))\|x)) = ((x\|y)\|(x\|y))$	[75 → 487]
1107	$(x\|(y\|(x\|x))) = (x\|x)$	[217 → 193,76,76,490,16]
1112,1111	$(((x\|y)\|(x\|y))\|x) = (x\|y)$	[201 → 193,516,76,62,76]
1114,1113	$((x\|x)\|(y\|x)) = x$	[199 → 193,1112,504,76,36,76]
1116,1115	$(x\|((x\|x)\|y)) = (x\|x)$	[197 → 193,1114,1112,1114,482,76]
1118,1117	$((x\|x)\|(x\|y)) = x$	[125 → 193,1112,488,1114,36,1114]

1119	$((x\|y)\|((((((x\|x)\|x)\|y)\|y)\|(x\|x))\|(z\|(x\|x)))) = x$	[169 → 193,1118,1118,1118,1118,1118,62,76,1118]
1824,1823	$((((((x\|x)\|x)\|y)\|y)\|(x\|x))\|(((((x\|x)\|x)\|y)\|y)\|(x\|x))) = (x\|x)$	[1119 → 1107,62]
1830,1829	$(((((x\|x)\|x)\|y)\|y)\|(x\|x)) = x$	[1119 → 179,1824,1824,76,1824,76,1824,76,1118]
1834,1833	$((((x\|(x\|x))\|y)\|y)\|x) = (x\|x)$	[1117 → 1829,1118]
1835	$((x\|(x\|x))\|(((x\|x)\|x)\|((x\|x)\|x))) = ((x\|x)\|x)$	[1829 → 1829]
1856,1855	$((((((x\|x)\|x)\|y)\|y)\|((((x\|x)\|x)\|y)\|y))\|x) = ((((x\|x)\|x)\|y)\|y)$	[1829 → 1117]
1859	$((x\|x)\|((((x\|x)\|x)\|y)\|y)) = x$	[1829 → 1111,1830,1830]
1886,1885	$(x\|(((x\|(x\|x))\|y)\|y)) = (x\|x)$	[1833 → 1111,1834,1118,1834]
1899	$(x\|(((((x\|x)\|x)\|y)\|y)\|((((x\|x)\|x)\|((((x\|x)\|x)\|y)\|y))\|y))) = ((((x\|x)\|x)\|y)\|y)$	[1859 → 129,1856]
1961	$(x\|(((x\|x)\|x)\|(((x\|x)\|x)\|((x\|x)\|x)))) = (x\|x)$	[1835 → 1885]
1964,1963	$((((x\|x)\|x)\|(((x\|x)\|x)\|((x\|x)\|x)))\|x) = (x\|x)$	[1835 → 1833]
2005	$(((x\|x)\|x)\|(((x\|x)\|x)\|((x\|x)\|x))) = ((x\|x)\|x)$	[1961 → 449,1964,1964,1118,1964,382,1116,16]
2042,2041	$(((x\|x)\|x)\|((((x\|x)\|x)\|y)\|y)) = (((x\|x)\|x)\|((x\|x)\|x))$	[2005 → 1885]
2044,2043	$(((((x\|x)\|x)\|y)\|y)\|((x\|x)\|x)) = (((x\|x)\|x)\|((x\|x)\|x))$	[2005 → 1833]
2050,2049	$((((x\|x)\|x)\|((x\|x)\|x))\|y) = ((x\|x)\|x)$	[2005 → 181,1886,88]
2063	$((((x\|x)\|x)\|y)\|y) = ((x\|x)\|x)$	[1899:2042,2050,2044,90]
2255	$(((x\|x)\|x)\|(y\|y)) = y$	[2063 → 15]
2769	$(x\|(x\|x)) = ((y\|y)\|y)$	[2255 → 2063]
2770	$\$Ans(ONE_SS)$	[2769,1]

Proof MOL-Sh-part2

2	$((y\|x)\|(((x\|x)\|z)\|(((((x\|y)\|z)\|z)\|x)\|(x\|u)))) = x$	[]
3	$(x\|(x\|x)) = 1$	[]
4	$((x\|x)\|x) = 1$	[]
5	$(A\|((B\|C)\|(B\|C))) \neq (B\|((A\|C)\|(A\|C)))$ | $\$Ans(A_SS)$	[]
6	$((A\|A)\|(A\|B)) \neq A$ | $\$Ans(B_SS)$	[]
7	$(A\|(B\|(A\|(C\|C)))) \neq (A\|(C\|(A\|(B\|B)))$ | $\$Ans(MOD_SS)$	[]
258	$(x\|(x\|x)) = (y\|(y\|y))$	[3 → 3]
259	$(x\|(x\|x)) = ((y\|y)\|y)$	[4 → 3]
260	$((x\|y)\|(y\|(((((y\|x)\|(((y\|y)\|z)\|(((((y\|y)\|z)\|z)\|y)\|(y\|u))))\|(((y\|y)\|z)\|(((((y\|y)\|z)\|z)\|y)\|(y\|u))))\|y)\|(y\|v)))) = y$	[2 → 2]
261	$((x\|(y\|z))\|((((y\|z)\|(y\|z))\|u)\|((((((y\|z)\|x)\|u)\|u)\|(y\|z))\|z))) = (y\|z)$	[2 → 2]
262	$((x\|y)\|(y\|y)) = y$	[260 → 260]
263	$((x\|y)\|(((y\|y)\|z)\|y)) = y$	[260 → 2]
264	$((x\|(x\|x))\|(y\|y)) = y$	[259 → 262]
265	$((x\|(y\|y))\|y) = (y\|y)$	[262 → 262]
266	$(((x\|x)\|(y\|x))\|((((y\|x)\|(y\|x))\|z)\|((((x\|z)\|z)\|(y\|x))\|x))) = (y\|x)$	[262 → 261]
267	$(((x\|x)\|(y\|x))\|((((y\|x)\|(y\|x))\|z)\|((((x\|z)\|z)\|(y\|x))\|((y\|x)\|u)))) = (y\|x)$	[262 → 2]
268	$((x\|(x\|x))\|y) = (y\|y)$	[262 → 264]
269	$((x\|y)\|(((y\|y)\|((y\|x)\|(y\|x)))\|(((y\|x)\|y)\|(y\|z)))) = y$	[264 → 2]
270	$(1\|x) = (x\|x)$	[4 → 265]
271	$((x\|y)\|((z\|(y\|y))\|y)) = y$	[265 → 262]
272	$((x\|(((y\|y)\|z)\|y))\|y) = (((y\|y)\|z)\|y)$	[263 → 263]
273	$((x\|(y\|y))\|((z\|y)\|(y\|y))) = (y\|y)$	[262 → 271]
274	$(((x\|x)\|y)\|x) = (x\|x)$	[263 → 272]
275	$(((1\|x)\|y)\|x) = (x\|x)$	[270 → 274]
276	$((x\|y)\|(x\|x)) = ((x\|x)\|(x\|x))$	[262 → 274]
277	$((x\|x)\|(((x\|x)\|x)\|(((x\|x)\|x)\|(x\|y)))) = x$	[274 → 2]
279	$((x\|y)\|(x\|x)) = x$	[262 → 276]

280	$(x\|((x\|y)\|(x\|y))) = (x\|y)$	[279 → 279]
281	$(x\|((y\|x)\|(y\|x))) = (y\|x)$	[271 → 279]
282	$((x\|y)\|(1\|x)) = x$	[270 → 279]
283	$((x\|y)\|((z\|(x\|x))\|x)) = x$	[265 → 279]
284	$(x\|(1\|(x\|y))) = (x\|y)$	[282 → 282]
285	$(x\|(1\|(y\|x))) = (y\|x)$	[271 → 282]
286	$(x\|(((y\|y)\|y)\|(z\|x))) = (z\|x)$	[4 → 285]
287	$(x\|((y\|(y\|y))\|(z\|x))) = (z\|x)$	[3 → 285]
288	$((x\|y)\|(((y\|y)\|z)\|(((((y\|x)\|z)\|z)\|y)\|(u\|y)))) = y$	[285 → 2]
289	$((((x\|y)\|(x\|y))\|x)\|(((x\|x)\|z)\|(((((x\|y)\|z)\|z)\|x)\|(x\|u)))) = x$	[280 → 2]
290	$(x\|((((y\|x)\|(y\|x))\|z)\|(y\|x))) = (y\|x)$	[274 → 281]
291	$(x\|x) = (1\|x)$	[282 → 287]
292	$((x\|x)\|((y\|(y\|y))\|(((x\|x)\|x)\|(x\|z)))) = x$	[259 → 277]
293	$((x\|x)\|(((x\|x)\|x)\|(((x\|x)\|x)\|(y\|x)))) = x$	[290 → 277]
295	$((x\|x)\|((y\|(y\|y))\|((z\|(z\|z))\|(x\|u)))) = x$	[259 → 292]
296	$((x\|x)\|(y\|(y\|y))) = x$	[258 → 293]
297	$((x\|x), 1) = x$	[3 → 293]
298	$(x, 1) = (x\|x)$	[279 → 297]
299	$(x\|(1, 1)) = 1$	[297 → 262]
300	$(x\|(y\|(y\|y))) = (x\|x)$	[3 → 298]
301	$(((1, 1)\|x)\|((1, 1)\|x)) = (1, 1)$	[275 → 298]
302	$((x\|(y\|(y\|y)))\|(z\|(z\|z))) = x$	[300 → 296]
303	$((1, 1)\|(1, 1)) = ((1, 1)\|x)$	[301 → 280]
305	$((1, 1)\|x) = 1$	[299 → 303]
308	$((x\|x)\|(x\|y)) = x$	[283 → 295]
309	$\$Ans(B_SS)$	[308,6]
310	$(x\|((x\|x)\|y)) = (x\|x)$	[308 → 308]
311	$((x\|(y\|(y\|y)))\|(x\|z)) = x$	[300 → 308]
312	$((x, 1)\|(x\|y)) = x$	[298 → 308]
313	$((1\|x)\|(x\|y)) = x$	[291 → 308]
314	$((x\|x)\|(y\|x)) = x$	[290 → 308]
315	$(((x\|y)\|(x\|y))\|x) = (x\|y)$	[283 → 308]
316	$(x\|x) = (x, 1)$	[312 → 280]
317	$((1\|(x\|y))\|x) = (x\|y)$	[283 → 313]
318	$(x\|(y\|(x\|x))) = (x\|x)$	[314 → 314]
319	$((x\|(y\|(y\|y)))\|(z\|x)) = x$	[300 → 314]
320	$((x\|x)\|(x\|y)) = ((x\|x)\|(x\|x))$	[314 → 310]
321	$((x\|y)\|(y\|((y\|y)\|z))) = y$	[310 → 262]
322	$((x\|((x\|x)\|y))\|(x\|z)) = x$	[310 → 308]
323	$(((x\|(x\|x))\|(y\|z))\|y) = (y\|z)$	[3 → 317]
324	$(x\|(y\|((z\|(z\|z))\|x))) = (x\|x)$	[268 → 318]
325	$(1\|(x\|((x\|x)\|y))) = x$	[305 → 321]
326	$(1\|(x\|(y\|(x\|x)))) = x$	[290 → 325]
327	$(1\|((x\|x)\|(y\|x))) = (x\|x)$	[314 → 326]
328	$(((x\|x)\|x)\|((y\|y)\|(z\|y))) = (y\|y)$	[4 → 327]
329	$((x\|x)\|(x\|y)) = ((x\|x)\|(x\|z))$	[320 → 320]
331	$((x\|y)\|(y\|(((((y\|x)\|(z\|y))\|(z\|y))\|y)\|(u\|y)))) = y$	[314 → 288]
332	$((x\|y)\|(y\|(((((y\|x)\|(y\|z))\|(y\|z))\|y)\|(u\|y)))) = y$	[308 → 288]
333	$((x\|y)\|(1\|(((((y\|x)\|y)\|y)\|y)\|(z\|y)))) = y$	[4 → 288]
334	$(((((x\|y)\|x)\|x)\|x)\|(y\|x)) = x$	[286 → 288]
335	$((x\|y)\|((((y\|x)\|(z\|y))\|(z\|y))\|y)) = y$	[281 → 331]
336	$((x\|y)\|((((x\|((x\|y)\|(x\|y)))\|(z\|x))\|(z\|x))\|x)) = x$	[315 → 335]
337	$((x\|y)\|((((y\|x)\|(y\|z))\|(y\|z))\|y)) = y$	[281 → 332]
338	$((x\|y)\|((((x\|((x\|y)\|(x\|y)))\|(x\|z))\|(x\|z))\|x)) = x$	[315 → 337]

339	$(x\|(((((y\|x)\|(x\|x))\|((y\|x)\|z))\|((y\|x)\|z))\|(y\|x))) = (y\|x)$	[314 → 337]
340	$(x\|(((((x\|y)\|(x\|x))\|((x\|y)\|z))\|((x\|y)\|z))\|(x\|y))) = (x\|y)$	[308 → 337]
341	$((x\|y)\|((((x\|y)\|(z\|x))\|(z\|x))\|x)) = x$	[280 → 336]
342	$((x\|y)\|((((x\|y)\|(x\|z))\|(x\|z))\|x)) = x$	[280 → 338]
343	$((x\|y)\|((x\|y)\|x)) = x$	[302 → 342]
344	$((x,1)\|(1\|(((1\|x),1)\|(1\|y)))) = 1$	[305 → 269]
345	$((x,1)\|(1\|(((1\|x),1)\|y))) = 1$	[326 → 344]
346	$((x,1)\|(1\|(((x\|x),1)\|y))) = 1$	[291 → 345]
347	$((x,1)\|(1\|(x\|y))) = 1$	[297 → 346]
348	$((x\|x)\|(1\|(x\|y))) = 1$	[316 → 347]
349	$((x\|x)\|((x\|y)\|(x\|y))) = 1$	[291 → 348]
350	$(((x\|y)\|(x\|y))\|(y\|y)) = 1$	[273 → 349]
352	$(x\|(((x\|((y\|x)\|z))\|((y\|x)\|z))\|(y\|x))) = (y\|x)$	[262 → 339]
353	$(x\|(((x\|y)\|((y\|x)\|((((y\|x)\|(z\|y))\|(z\|y))\|y)))\|(y\|x))) = (y\|x)$	[341 → 352]
354	$(x\|(x\|(y\|x))) = (y\|x)$	[302 → 352]
355	$(x\|(((x\|((x\|y)\|z))\|((x\|y)\|z))\|(x\|y))) = (x\|y)$	[279 → 340]
356	$(x\|(x\|(x\|y))) = (x\|y)$	[302 → 355]
357	$((x\|y)\|(((x\|x)\|z)\|(((((x\|y)\|z)\|z)\|x)\|(x\|u)))) = x$	[315 → 289]
358	$((x\|y)\|(((x\|x),1)\|(((((x\|y)\|(x\|y)),1)\|x)\|(x\|z)))) = x$	[316 → 357]
359	$((x\|y)\|(x\|(((((x\|y)\|(x\|y)),1)\|x)\|(x\|z)))) = x$	[297 → 358]
360	$((x\|y)\|(x\|(((x\|y)\|x)\|(x\|z)))) = x$	[297 → 359]
361	$(x\|(((x\|y)\|y)\|(y\|x))) = (y\|x)$	[342 → 353]
362	$(((x\|x)\|(y\|x))\|((((y\|x)\|(y\|x))\|(x\|x))\|((x\|(y\|x))\|x))) = (y\|x)$	[279 → 266]
363	$(x\|((((y\|x)\|(y\|x))\|(x\|x))\|((x\|(y\|x))\|x))) = (y\|x)$	[314 → 362]
364	$(x\|(1\|((x\|(y\|x))\|x))) = (y\|x)$	[350 → 363]
366	$((x\|(y\|x))\|x) = (y\|x)$	[285 → 364]
367	$(((x\|(x\|x))\|(y\|z))\|y) = ((y\|(y\|z))\|y)$	[323 → 366]
368	$((x\|(y\|x))\|(x\|((y\|x)\|(x\|z)))) = x$	[366 → 360]
369	$((x\|(y\|x))\|(y\|x)) = x$	[366 → 343]
373	$((x\|(x\|y))\|x) = (x\|y)$	[323 → 367]
374	$((x\|y)\|x) = (x\|(x\|y))$	[356 → 373]
375	$((x\|y)\|y) = (y\|(x\|y))$	[354 → 373]
376	$((x\|(x\|y))\|(x\|y)) = x$	[373 → 343]
377	$(((x\|y)\|x)\|x) = (x\|y)$	[374 → 373]
378	$(x\|((y\|(x\|y))\|(y\|x))) = (y\|x)$	[375 → 361]
380	$((x\|y)\|(((y\|y)\|((y\|x)\|z))\|(((y\|x)\|y)\|(y\|u)))) = y$	[376 → 2]
382	$((x\|y)\|(1\|(((y\|x)\|y)\|(z\|y)))) = y$	[377 → 333]
383	$(((x\|y)\|x)\|(y\|x)) = x$	[377 → 334]
386	$(((x\|(x\|(x\|y)))\|x)\|(y\|x)) = x$	[356 → 383]
387	$((x\|(x\|y))\|(y\|x)) = x$	[374 → 383]
389	$((x\|y)\|(1\|((y\|(y\|x))\|(z\|y)))) = y$	[374 → 382]
390	$((x\|y)\|(1\|((x\|(x\|(1\|(x\|y))))\|(z\|x)))) = x$	[317 → 389]
391	$((x\|y)\|(1\|((y\|(y\|x))\|(y\|z)))) = y$	[377 → 389]
392	$((1\|((x\|(x\|y))\|(x\|z)))\|(((y\|x)\|(1\|((x\|(x\|y))\|(x\|z))))\|((y\|x)\|(1\|((x\|(x\|y))\|(x\|z))))))) = x$	[281 → 391]
393	$((x\|y)\|(1\|((x\|(x\|y))\|(z\|x)))) = x$	[284 → 390]
395	$((x\|x)\|(((x\|x)\|y)\|(((x\|x)\|x)\|(x\|z)))) = x$	[356 → 380]
397	$((x\|x)\|(((x\|x)\|y)\|((z\|(z\|z))\|(x\|u)))) = x$	[259 → 395]
400	$(x\|((((y\|x)\|(y\|x))\|z)\|((((x\|z)\|z)\|(y\|x))\|((y\|x)\|u)))) = (y\|x)$	[314 → 267]
401	$(x\|((y\|x)\|((((x\|y)\|y)\|(y\|x))\|((y\|x)\|z)))) = (y\|x)$	[315 → 400]
402	$((1\|((x\|(x\|y))\|(x\|z)))\|(x\|((y\|x)\|(1\|((x\|(x\|y))\|(x\|z))))))) = x$	[391 → 392]
403	$((1\|((x\|(x\|y))\|(x\|z)))\|(x\|x)) = x$	[391 → 402]
405	$((1\|((x\|y)\|(y\|z)))\|(y\|y)) = y$	[354 → 403]
406	$((1\|((x\|y)\|(z\|y)))\|(y\|y)) = y$	[401 → 405]

407	$(((x\|y)\|(y\|z))\|((x\|y)\|(y\|z))) = (y\|((x\|y)\|(y\|z)))$	[405 → 275]
408	$((1\|((x\|y)\|(z\|x)))\|(x\|x)) = x$	[377 → 406]
409	$(((x\|y)\|(z\|x))\|((x\|y)\|(z\|x))) = (x\|((x\|y)\|(z\|x)))$	[408 → 275]
410	$((x\|y)\|(x\|((z\|x)\|(x\|y)))) = ((z\|x)\|(x\|y))$	[407 → 281]
411	$((x\|((y\|z)\|(z\|u)))\|(z\|((y\|z)\|(z\|u)))) = ((y\|z)\|(z\|u))$	[407 → 262]
412	$(x\|((x\|y)\|((z\|(x\|y))\|((x\|y)\|(1\|((x\|(x\|y))\|(u\|x)))))))) = ((z\|(x\|y))\|((x\|y)\|(1\|((x\|(x\|y))\|(u\|x)))))$	[393 → 410]
414	$((x\|y)\|(y\|((y\|z)\|(x\|y)))) = ((y\|z)\|(x\|y))$	[409 → 281]
417	$((x\|((y\|((z\|u)\|(u\|v)))\|(u\|((z\|u)\|(u\|v)))))\|(u\|((z\|u)\|(u\|v)))) = ((z\|u)\|(u\|v))$	[411 → 411]
420	$((x\|(y\|z))\|((y\|z)\|(1\|((y\|(y\|z))\|(u\|y))))) = (y\|((y\|z)\|((x\|(y\|z))\|y)))$	[393 → 412]
421	$(x\|((x\|y)\|((z\|(x\|y))\|x))) = ((z\|(x\|y))\|x)$	[393 → 420]
422	$(((x\|(x\|y))\|(y\|x))\|y) = (y\|((y\|x)\|(x\|y)))$	[387 → 421]
423	$(x\|((x\|y)\|(y\|x))) = (y\|x)$	[387 → 422]
424	$((x\|y)\|(x\|y)) = ((y\|x)\|(x\|y))$	[423 → 414]
425	$(x\|((y\|x)\|(x\|y))) = (x\|y)$	[424 → 280]
426	$(((x\|y)\|(y\|x))\|((y\|x)\|z)) = (y\|x)$	[424 → 308]
427	$((x\|(y\|x))\|(x\|y)) = x$	[425 → 368]
428	$(((x\|y)\|y)\|(y\|x)) = y$	[375 → 427]
429	$(x\|y) = (y\|x)$	[427 → 378]
430	$((x\|y)\|(((y\|y)\|z)\|(((((x\|y)\|z)\|z)\|y)\|(y\|u)))) = y$	[429 → 2]
431	$(x\|((y\|x)\|(y\|((y\|x)\|z)))) = (y\|x)$	[428 → 401]
432	$(((x\|y)\|(y\|x))\|((x\|y)\|z)) = (y\|x)$	[429 → 426]
433	$(x\|((y\|x)\|(y\|(z\|(y\|x))))) = (y\|x)$	[431 → 431]
434	$((((x\|y)\|z)\|(u\|(u\|u)))\|(y\|x)) = ((x\|y)\|z)$	[432 → 319]
435	$(x\|((y\|z)\|(z\|x))) = (z\|x)$	[410 → 433]
439	$(x\|((y\|z)\|(x\|z))) = (z\|x)$	[429 → 435]
440	$(x\|((y\|y)\|(y\|z))) = (y\|x)$	[329 → 435]
441	$(x\|(y\|(x\|(z\|y)))) = ((z\|y)\|x)$	[386 → 439]
442	$(x\|((x\|y)\|(z\|y))) = (y\|x)$	[429 → 439]
445	$(x\|((x\|(y\|z))\|z)) = ((y\|z)\|x)$	[386 → 442]
446	$(((x\|y)\|(z\|y))\|x) = (y\|x)$	[429 → 442]
448	$(((x\|(y\|z))\|z)\|x) = ((y\|z)\|x)$	[386 → 446]
451	$(((x\|(x\|x))\|y)\|z) = (z\|(y\|y))$	[324 → 441]
452	$(x\|(y\|(x\|(z\|y)))) = (x\|(z\|y))$	[429 → 441]
453	$((x\|((y\|(y\|y))\|(z\|u)))\|(z\|z)) = z$	[397 → 445]
454	$((((x\|y)\|z)\|y)\|z) = ((x\|y)\|z)$	[429 → 448]
455	$(((x\|(y\|z))\|z)\|x) = ((z\|y)\|x)$	[429 → 448]
456	$((1\|x)\|y) = (y\|(x\|x))$	[349 → 451]
457	$((x, 1)\|y) = (y\|(x\|x))$	[429 → 456]
458	$((1\|x)\|y) = ((x\|x)\|y)$	[429 → 456]
459	$(x\|((y\|(z\|(z\|z)))\|(y\|u))) = (x\|((y\|u)\|(x\|y)))$	[311 → 452]
460	$((x\|(y\|(z\|x)))\|y) = (y\|(z\|x))$	[429 → 452]
461	$((x\|(y\|(z\|x)))\|y) = ((x\|z)\|y)$	[429 → 455]
463	$(x\|(y\|z)) = ((z\|y)\|x)$	[460 → 461]
464	$((x\|y)\|z) = (z\|(y\|x))$	[461 → 460]
465	$(x\|(y\|(z\|u))) = ((y\|(u\|z))\|x)$	[463 → 463]
466	$(x\|(y\|(z, 1))) = ((y\|(z\|z))\|x)$	[457 → 463]
467	$(x\|(y\|z)) = (x\|(z\|y))$	[429 → 463]
468	$(x\|((y\|x)\|(y\|z))) = (y\|x)$	[463 → 435]
469	$(((x\|y)\|(x\|y))\|(z\|(y\|x))) = (x\|y)$	[463 → 308]
470	$((x\|(y\|z))\|u) = (u\|(x\|(z\|y)))$	[464 → 464]
471	$(((1\|x)\|y)\|z) = (z\|(y\|(x\|x)))$	[458 → 464]
472	$(((x\|y)\|z)\|u) = (u\|(z\|(y\|x)))$	[467 → 464]
473	$((x\|(y\|z))\|u) = (u\|((z\|y)\|x))$	[467 → 464]

474	$(x\|(((y\|z)\|x)\|(u\|(z\|y)))) = ((y\|z)\|x)$	[464 → 468]
477	$(((x\|(y\|z))\|u)\|v) = (v\|((x\|(z\|y))\|u))$	[465 → 464]
478	$((x\|((y\|z)\|(z\|u)))\|(z\|z)) = z$	[417 → 453]
481	$((x\|((y\|z)\|(y\|u)))\|(y\|y)) = y$	[429 → 478]
484	$((((x\|y)\|(z\|y))\|u)\|(y\|y)) = y$	[472 → 478]
486	$(((x\|x)\|y)\|((z\|x)\|(z\|x))) = (z\|x)$	[328 → 484]
488	$((x\|x)\|(y\|((z\|x)\|(u\|x)))) = x$	[472 → 484]
490	$(((x\|(y\|z))\|(x\|(y\|z)))\|(u\|((z\|y)\|(v\|(x\|(y\|z))))))) = (x\|(y\|z))$	[469 → 488]
492	$(x\|((((y\|y)\|z)\|x)\|(u\|y))) = (((y\|y)\|z)\|x)$	[486 → 468]
493	$((((x\|((y\|z)\|(y\|u)))\|(y\|y))\|(v\|(v\|v)))\|(((y\|z)\|(y\|u))\|x)) = y$	[481 → 434]
494	$(((x\|x)\|y)\|((x\|z)\|(x\|z))) = (x\|z)$	[469 → 492]
495	$(x\|(((y\|y)\|z)\|((y\|u)\|x))) = ((y\|u)\|x)$	[492 → 474]
496	$((((x\|x)\|y)\|(((x\|x)\|y)\|((x\|z)\|(x\|z))))\|(x\|z)) = ((x\|x)\|y)$	[494 → 376]
497	$(x\|((y\|z)\|(x\|y))) = (x\|y)$	[322 → 459]
498	$(((x\|y)\|x)\|(((y\|x)\|z)\|x)) = (((x\|y)\|x)\|(y\|x))$	[383 → 497]
499	$(x\|((x\|y)\|(y\|z))) = (x\|y)$	[429 → 497]
500	$(((x\|y)\|(y\|z))\|x) = (x\|y)$	[464 → 497]
502	$((((x\|y)\|z)\|(z\|u))\|(y\|x)) = ((x\|y)\|z)$	[464 → 499]
503	$(((x\|y)\|x)\|(((y\|x)\|z)\|x)) = x$	[383 → 498]
506	$((((x\|x)\|y)\|(x\|z))\|(x\|z)) = ((x\|x)\|y)$	[494 → 496]
508	$((x\|((y\|x)\|(x\|z)))\|(u\|((z\|x)\|(v\|((y\|x)\|(x\|z))))))) = ((y\|x)\|(x\|z))$	[407 → 490]
509	$((x\|(((((y\|x)\|z)\|z)\|x)\|(x\|y)))\|(u\|x)) = (((((y\|x)\|z)\|z)\|x)\|(x\|y))$	[430 → 508]
510	$(((((x\|y)\|z)\|z)\|y)\|(y\|x)) = y$	[478 → 509]
511	$(((x\|(x\|(y\|z)))\|z)\|(z\|y)) = z$	[464 → 510]
512	$((x\|(y\|(y\|(z\|x))))\|(x\|z)) = x$	[472 → 510]
513	$(((x\|(x\|(y\|z)))\|z)\|(y\|z)) = z$	[429 → 511]
514	$((x\|y)\|((z\|(z\|(x\|y)))\|y)) = y$	[464 → 511]
516	$(((x\|((y\|z)\|x))\|y)\|(z\|y)) = y$	[464 → 513]
518	$(x\|((y\|(y\|((x\|(z\|(z\|(u\|x))))\|(x\|u))))\|(x\|u))) = (x\|u)$	[512 → 514]
519	$((x\|y)\|((z\|((y\|x)\|z))\|y)) = y$	[464 → 514]
520	$((x\|y)\|((z\|((u\|((y\|x)\|u))\|y))\|((x\|y)\|((u\|((y\|x)\|u))\|y)))) = y$	[439 → 516]
521	$(x\|((y\|(y\|x))\|(x\|z))) = (x\|z)$	[512 → 518]
523	$((x\|y)\|((z\|((u\|((y\|x)\|u))\|y))\|y)) = y$	[519 → 520]
524	$((x\|((y\|((z\|(x\|x))\|y))\|z))\|z) = (z\|z)$	[523 → 495]
525	$((((x\|(x\|(y\|(z\|z))))\|y)\|z)\|y) = (y\|y)$	[477 → 524]
526	$(((x\|((y\|(z\|z))\|x))\|y)\|z) = ((y\|y)\|z)$	[524 → 448]
527	$(((x\|(x\|(y\|(z\|z))))\|y)\|z) = ((y\|y)\|z)$	[525 → 454]
528	$(((x\|((y\|(z\|z))\|x))\|((y\|(z\|z))\|x))\|z) = ((((y\|(z\|z))\|x)\|((y\|(z\|z))\|x))\|z)$	[454 → 526]
529	$((x\|(x\|(y\|y)))\|y) = ((x\|x)\|y)$	[500 → 526]
530	$((x\|(((y\|(y\|y))\|z)\|x))\|z) = ((x\|x)\|z)$	[451 → 529]
531	$((((((x\|x)\|y)\|(x\|x))\|(x\|x))\|((y\|(x\|x))\|z))\|x) = ((((y\|(x\|x))\|z)\|((y\|(x\|x))\|z))\|x)$	[503 → 527]
532	$((((x\|(y\|y))\|z)\|((x\|(y\|y))\|z))\|y) = (z\|y)$	[369 → 528]
533	$((((x\|(y\|y))\|z)\|((x\|(y\|y))\|z))\|y) = ((((y\|y)\|x)\|((x\|(y\|y))\|z))\|y)$	[506 → 531]
534	$((((x\|x)\|y)\|((y\|(x\|x))\|z))\|x) = (z\|x)$	[532 → 533]
536	$((((x\|x)\|y)\|(z\|(y\|(x\|x))))\|x) = (z\|x)$	[470 → 534]
537	$((((x\|x)\|y)\|(z\|(y\|(x\|x))))\|x) = (((z\|(y\|(x\|x)))\|(z\|u))\|x)$	[468 → 534]
539	$(((x\|(y\|(z\|z)))\|(x\|u))\|z) = (x\|z)$	[536 → 537]
540	$(((x\|(y\|z))\|(x\|u))\|(z\|z)) = (x\|(z\|z))$	[314 → 539]
541	$(((x\|y)\|(x\|z))\|((u\|y)\|(v\|y))) = (x\|((u\|y)\|(v\|y)))$	[488 → 539]
542	$(((x\|y)\|(x\|((z\|z)\|u)))\|z) = (x\|z)$	[470 → 539]
544	$(((x\|y)\|(x\|(z\|u)))\|(z\|z)) = (x\|(z\|z))$	[314 → 542]
545	$((x\|x)\|((y\|z)\|(z\|(u\|x)))) = (z\|(x\|x))$	[473 → 540]
546	$((x\|x)\|(((y\|x)\|(z\|u))\|z)) = ((z\|u)\|(x\|x))$	[502 → 545]
547	$((x\|x)\|((y\|z)\|(z\|(u\|x)))) = ((x\|x)\|z)$	[464 → 545]

548	$(((x,1)\|(x,1))\|((y\|z)\|((u\|(x\|x))\|z))) = (((x,1)\|(x,1))\|z)$	[466 → 547]
549	$(((x\|y)\|(z\|y))\|((u\|y)\|(u\|v))) = (u\|((z\|y)\|(x\|y)))$	[464 → 541]
550	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[541 → 549]
551	$\$Ans(A_SS)$	[550,5]
552	$(x\|((y\|z)\|((u\|(x\|x))\|z))) = (((x,1)\|(x,1))\|z)$	[312 → 548]
553	$(x\|((y\|z)\|((u\|(x\|x))\|z))) = (x\|z)$	[312 → 552]
554	$(((x\|y)\|(x\|z))\|((u\|v)\|(x\|v))) = (((x\|y)\|(x\|z))\|v)$	[493 → 553]
555	$(((x\|y)\|(x\|((x\|z)\|u)))\|z) = (x\|((x\|z)\|(x\|z)))$	[544 → 554]
556	$(((x\|y)\|(x\|((x\|z)\|u)))\|z) = (z\|x)$	[442 → 555]
559	$((x\|(x\|y))\|(y\|((y\|x)\|z))) = ((x\|x)\|(y\|((y\|x)\|z)))$	[556 → 530]
560	$(x\|((y\|y)\|(x\|((x\|y)\|z)))) = (x\|((x\|y)\|z))$	[559 → 521]
561	$(x\|((y\|y)\|(((x\|y)\|z)\|x))) = (x\|((x\|y)\|z))$	[429 → 560]
562	$(x\|((x\|y)\|(z\|z))) = (x\|((x\|z)\|(x\|y)))$	[546 → 561]
563	$(x\|((x\|(y\|y))\|(x\|z))) = (x\|(y\|(x\|z)))$	[440 → 562]
564	$(x\|(((1\|y)\|x)\|(x\|z))) = (x\|(y\|(x\|z)))$	[456 → 563]
565	$(x\|(((1\|y)\|x)\|(x\|(z\|z)))) = (x\|(z\|(x\|(y\|y))))$	[471 → 563]
566	$(x\|(y\|(x\|(z\|z)))) = (x\|(z\|(x\|(y\|y))))$	[564 → 565]
567	$\$Ans(MOD_SS)$	[566,7]

Proof MOL-Sh-sound

1	$x = x$	[]
2	$(x\|((y\|z)\|(y\|z))) = (y\|((x\|z)\|(x\|z)))$	[]
4,3	$((x\|x)\|(x\|y)) = x$	[]
6	$(x\|(y\|(x\|(z\|z)))) = (x\|(z\|(x\|(y\|y))))$	[]
7	$(x\|(x\|x)) = 1$	[]
9	$((B\|A)\|(((A\|A)\|C)\|(((((A\|B)\|C)\|C)\|A)\|(A\|D)))) \neq A \mid \$Ans(MOL_Sh)$	[]
11,10	$(x\|((x\|x)\|y)) = (x\|x)$	[3 → 3]
12	$((x\|x)\|(y\|((x\|z)\|(x\|z)))) = x$	[2 → 3]
16	$(x\|((y\|x)\|(y\|x))) = (y\|x)$	[3 → 2]
18	$((x\|x)\|((y\|(x\|z))\|(y\|(x\|z)))) = (y\|(x\|x))$	[3 → 2,4]
20	$((x\|x)\|x) = 1$	[3 → 7]
22	$(x\|(((x\|y)\|y)\|((x\|y)\|y))) = 1$	[2 → 7]
25,24	$((x\|x),1) = x$	[7 → 3]
28	$(x,1) = (x\|x)$	[7 → 10]
31	$(x\|x) = (x,1)$	[28]
34	$(x\|(y\|(x\|z))) = (x\|((z\|z)\|(x\|(y\|y))))$	[3 → 6]
36,35	$(x\|(y\|(x\|x))) = (x\|x)$	[10 → 6,4]
37	$(x\|(y,1)) = (x\|(x\|(x\|(y\|y))))$	[7 → 6]
38	$((x\|x)\|(y\|x)) = x$	[3 → 6,11,4]
41	$(x\|((y\|y)\|(x\|(z\|z)))) = (x\|(z\|(x\|y)))$	[34]
58	$(x\|(y\|(x\|(x\|(y\|y))))) = (x,1)$	[20 → 6,4]
66	$(x\|((y,1)\|(y\|y))) = (y\|((x,1)\|(x,1)))$	[28 → 2]
68	$(x\|((y,1)\|(y,1))) = (y\|((x,1)\|(x\|x)))$	[66]
73	$(x\|(y\|(x\|(z,1)))) = (x\|(z\|(x\|(y\|y))))$	[31 → 6]
85,84	$((x,1)\|(x\|y)) = x$	[31 → 3]
88	$(x\|y) = (y\|x)$	[68:85,85]
91,90	$((x\|y)\|(y\|y)) = y$	[38 → 88]
92	$(x\|x) = (1\|x)$	[28 → 88]
94,93	$(1\|(x\|x)) = x$	[24 → 88]
95	$(((x\|x)\|y)\|x) = (x\|x)$	[10 → 88]
100,99	$((x\|y)\|(x\|x)) = x$	[3 → 88]

101	$(x\|((y\|z)\|(y\|z))) = (((x\|z)\|(x\|z))\|y)$	[2 → 88]
102	$(1\|x) = (x\|x)$	[92]
103	$(((x\|y)\|(x\|y))\|z) = (x\|((z\|y)\|(z\|y)))$	[101]
104	$((((A\|A)\|C)\|(((((A\|B)\|C)\|C)\|A)\|(A\|D)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$	[88 → 9]
122	$((x,1)\|(y\|((x\|z)\|(x\|z)))) = x$	[31 → 12]
146	$(x\|((y\|z)\|(x\|y))) = (x\|y)$	[12 → 6,4]
160	$((x\|(y\|y))\|y) = (y\|y)$	[90 → 90]
162	$((x\|y)\|(y,1)) = y$	[31 → 90]
177,176	$((x\|y)\|(x,1)) = x$	[31 → 99]
178	$(x\|((((x\|y)\|z)\|y)\|(((x\|y)\|z)\|y))) = (x\|y)$	[2 → 99]
192	$(x\|((y\|x),1)) = (y\|x)$	[31 → 16]
207,206	$((x\|y)\|(1\|y)) = y$	[92 → 90]
208	$((x\|y)\|(1\|x)) = x$	[92 → 99]
218	$((1\|x)\|(y\|((x\|z)\|(x\|z)))) = x$	[92 → 12]
226	$((x\|x)\|(y\|(1\|(x\|z)))) = x$	[92 → 12]
232,231	$(1\|(1\|x)) = x$	[92 → 93]
234,233	$(1\|(x,1)) = x$	[31 → 93]
291	$((x\|(y\|z))\|(x\|(y\|y))) = ((x\|(y\|z))\|(x\|(y\|z)))$	[18 → 35]
296	$((x\|(y\|z))\|(x\|(y\|z))) = ((x\|(y\|z))\|(x\|(y\|y)))$	[291]
300,299	$((x\|y)\|(x\|y)) = (1\|(y\|x))$	[6 → 102,94,94,94]
302	$((x\|(y\|z))\|(x\|(y\|y))) = (1\|((y\|z)\|x))$	[296:300]
328	$((1\|x)\|(y\|(1\|(z\|x)))) = x$	[218:300]
335	$(x\|(1\|(y\|((x\|y)\|z)))) = (x\|y)$	[178:300]
362	$((x,1)\|(y\|(1\|(z\|x)))) = x$	[122:300]
369,368	$((1\|(x\|y))\|z) = (y\|(1\|(x\|z)))$	[103:300,300]
382	$(x\|(1\|(y\|(x\|y)))) = 1$	[22:300]
474	$(x\|((y\|z)\|(x\|z))) = (x\|z)$	[88 → 146]
478	$(x\|((y\|z)\|(y\|x))) = (x\|y)$	[88 → 146]
488	$(x\|((x\|y)\|(y\|z))) = (x\|y)$	[88 → 146]
501	$(x\|(x\|(x\|y))) = (x\|y)$	[24 → 37,4]
504,503	$((x\|y)\|((x\|y)\|y)) = y$	[162 → 37,91]
505	$((x\|y)\|((x\|y)\|x)) = x$	[95 → 37,300,232,300,232,300,232,300,232,100]
509	$(x\|(x\|(y\|x))) = (x\|y)$	[88 → 501]
525	$((x\|y)\|((y\|x)\|y)) = y$	[88 → 503]
531	$(((x\|y)\|y)\|(x\|y)) = y$	[88 → 503]
552	$(x\|((y\|y)\|(x\|z))) = (x\|((z\|z)\|(x\|y)))$	[99 → 41]
558	$((x\|y)\|((z\|z)\|x)) = ((x\|y)\|(x\|((x\|y)\|z)))$	[99 → 41]
570	$((x\|y)\|(x\|((x\|y)\|z))) = ((x\|y)\|((z\|z)\|x))$	[558]
591,590	$(x\|((y\|x)\|x)) = (x\|y)$	[88 → 509]
637,636	$((x\|y)\|(y\|x)) = ((x\|y),1)$	[160 → 58,4,4,4,591,4]
682	$((x\|y)\|(y\|(y\|x))) = y$	[88 → 525]
709,708	$(((x\|y)\|y)\|y) = (x\|y)$	[531 → 505]
939,938	$(((x\|y)\|z)\|(1\|(y\|x))) = (x\|y)$	[73 → 208,234,94,234]
1014	$((x\|y)\|((z\|((x\|y)\|y))\|y)) = y$	[503 → 474,504]
1022	$(x\|((y\|z)\|(z\|x))) = (x\|z)$	[88 → 474]
1109	$(x\|(y\|((z\|y)\|x))) = (x\|(z\|y))$	[682 → 478]
1119	$(((x\|y)\|(x\|z))\|((z\|u)\|(z\|x))) = (((x\|y)\|(x\|z))\|z)$	[478 → 478]
1137	$(x\|((y\|x)\|(y\|z))) = (x\|y)$	[88 → 478]
1146,1145	$(((x\|y)\|(x\|z))\|z) = (z\|x)$	[88 → 478]
1147	$(((x\|y)\|(x\|z))\|((z\|u)\|(z\|x))) = (z\|x)$	[1119:1146]
1166	$(x\|((y\|y)\|(x\|z))) = (x\|((z\|x)\|(x\|y)))$	[478 → 41]
1191	$(((x\|y)\|y)\|((x\|y)\|(y\|z))) = (x\|y)$	[708 → 488,709]
1673	$(x\|(((y\|z)\|x)\|z)) = (x\|(y\|z))$	[682 → 1137]
1728	$(((C\|(A\|A))\|(((((A\|B)\|C)\|C)\|A)\|(A\|D)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$	[88 → 104]

2276	$(((C\|(A\|A))\|(((((B\|A)\|C)\|C)\|A)\|(A\|D)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$	[88 → 1728]
3039	$((x\|(y\|y))\|y) = (y\|y)$	[35 → 192,25,36]
3306	$(((C\|(A\|A))\|(((((C\|(B\|A))\|C)\|A)\|(A\|D)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$	[88 → 2276]
3457	$((x\|(1\|(y\|z)))\|(y\|y)) = y$	[88 → 226]
4471,4470	$(((x\|y)\|(y\|z))\|y) = (1\|((y\|z)\|(x\|y)))$	[3039 → 302,4,4,4]
5717,5716	$((x\|((y\|x)\|z))\|(u\|(1\|(y\|x)))) = (1\|(x\|((y\|x)\|z)))$	[335 → 328,232]
6399	$(((C\|(A\|A))\|(((C\|(C\|(B\|A)))\|A)\|(A\|D)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$	[88 → 3306]
11420	$(x\|((y\|(x\|y))\|(x\|z))) = (x\|z)$	[382 → 552,25,300,234]
12749	$((x\|y)\|((z\|((x\|y)\|z))\|x)) = x$	[382 → 570,177,300,234]
13756	$(((C\|(A\|A))\|((A\|(C\|(C\|(B\|A))))\|(A\|D)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$	[88 → 6399]
19707	$(((C\|(A\|A))\|((A\|(C\|(C\|(B\|A))))\|(D\|A)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$	[88 → 13756]
21759,21758	$((x\|y)\|((z\|(y\|(x\|y)))\|y)) = y$	[88 → 1014]
24702,24701	$(((x\|y)\|(y\|z)), 1) = (y\|((x\|y)\|(y\|z)))$	[1109 → 1147,4471,939,637]
26279,26278	$(1\|(x\|((y\|x)\|(x\|z)))) = ((y\|x)\|(x\|z))$	[1191 → 362,24702,5717]
26698	$(((C\|(A\|A))\|((D\|A)\|(A\|(C\|(C\|(B\|A))))))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$	[88 → 19707]
31493	$((((D\|A)\|(A\|(C\|(C\|(B\|A)))))\|(C\|(A\|A)))\|(B\|A)) \neq A \mid \$Ans(MOL_Sh)$	[88 → 26698]
33323	$((((D\|A)\|(A\|(C\|(C\|(B\|A)))))\|(C\|(A\|A)))\|(A\|B)) \neq A \mid \$Ans(MOL_Sh)$	[88 → 31493]
46866	$(x\|((y\|(y\|x))\|(x\|z))) = (x\|z)$	[88 → 11420]
48054,48053	$((x\|y)\|((z\|(z\|(x\|y)))\|x)) = x$	[88 → 12749]
51292,51291	$((x\|y)\|((z\|(z\|(x\|y)))\|y)) = y$	[21758 → 46866,21759]
51357	$((x\|y)\|(y\|(z\|(z\|(x\|y))))) = y$	[1022 → 46866,51292]
51361	$((x\|y)\|(x\|(z\|(z\|(x\|y))))) = x$	[478 → 46866,48054]
65437,65436	$((x\|(y\|(y\|(z\|x))))\|(x\|z)) = x$	[51357 → 51361,591]
97556	$((x\|((y\|z)\|(z\|u)))\|(z\|z)) = z$	[1166 → 3457,26279]
97921,97920	$(((x\|y)\|(y\|z))\|(u\|(y\|y))) = (1\|((y\|z)\|(x\|y)))$	[97556 → 1673,4471]
98134	$A \neq A \mid \$Ans(MOL_Sh)$	[33323:97921,369,65437,207]
98135	$\$Ans(MOL_Sh)$	[98134,1]

4 The Computer Programs

See the primary paper [8].

5 Conclusion

See the primary paper [8].

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