Key Theorem in the Solution of the Robbins Problem

THEOREM. *Robbins* $\Longrightarrow_{\mathrm{AC}}$ $\exists c \exists d\ (c + d = c)$.

Outline of proof.

7	$\overline{\overline{\overline{x}+y}+\overline{x+y}} = y$	[Robbins Equation]
10	$\overline{\overline{\overline{x+y}+\overline{x}+y}+y} = \overline{x+y}$	[7 → 7]
11	$\overline{\overline{\overline{\overline{x}+y}+x+y}+y} = \overline{\overline{x}+y}$	[7 → 7]
29	$\overline{\overline{\overline{\overline{x}+y}+x+2y}+\overline{\overline{x}+y}} = y$	[11 → 7]
54	$\overline{\overline{\overline{\overline{\overline{x}+y}+x+2y}+\overline{\overline{x}+y}+z}+\overline{y+z}} = z$	[29 → 7]
217	$\overline{\overline{\overline{\overline{\overline{x}+y}+x+2y}+\overline{\overline{x}+y}+\overline{y+z}+z}+z} = \overline{y+z}$	[54 → 7]
674	$\overline{\overline{\overline{\overline{\overline{\overline{x}+y}+x+2y}+\overline{\overline{x}+y}+\overline{y+z}+z}+z+u}+\overline{\overline{y+z}+u}} = u$	[217 → 7]
6736	$\overline{\overline{\overline{\overline{3x}+x}+\overline{3x}}+\overline{\overline{\overline{3x}+x}+5x}} = \overline{\overline{3x}+x}$	[10 → 674]
8855	$\overline{\overline{\overline{3x}+x}+5x} = \overline{3x}$	[6736 → 7 :54]
8865	$\overline{\overline{\overline{\overline{3x}+x}+\overline{3x}+2x}+\overline{3x}} = \overline{\overline{3x}+x}+2x$	[8855 → 7]
8866	$\overline{\overline{\overline{3x}+x}+\overline{3x}} = x$	[8855 → 7: 11]
8870	$\overline{\overline{\overline{\overline{3x}+x}+\overline{3x}+y}+\overline{x+y}} = y$	[8866 → 7]
8871	$\overline{\overline{3x}+x}+2x = 2x$	[8865 :8870]

Equation 8871 asserts the existence of an object c, namely $2x$, and an object d, namely $\overline{\overline{3x} + x}$, such that $c + d = c$. Q.E.D.