# The Mule and the Thoroughbred

Subspace iteration

- Generalization of the power method
- Very reliable
- Plods toward an answer
- Potentially high storage requirements
- Applicable to nonsymmetric systems
- Amenable to black box implementation

Lanczos

- Based on Krylov sequences
- Unreliable unless carefully implemented
- Fast convergence
- Low storage requirement
- Symmetric matrices only
- Black box–well, maybe

## Today's Paradigm

- We are given two spaces $\mathcal{V}$ and $\mathcal{W}$.
- We choose a "Ritz" pair $(\theta, x)$ such that

  1. $x \in \mathcal{V}$
  2. $(Ax - \theta x) \perp \mathcal{W}$

- $\mathcal{V}$ and $\mathcal{W}$ are expanded and enhanced.
- When the spaces get to big, they are purged and the process restarted.

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- The mule is implicitly restarted Arnoldi.
- The thoroughbreds are still in training.

## Ritz values

- A natural choice of the constraint subspace is $\mathcal{W} = \mathcal{V}$ (e.g., Arnoldi).
- Another choice is $\mathcal{W} = (A - \tau I)\mathcal{V}$. In the symmetric case he resulting values are related to Lehmann's optimal bounds.
- For $\tau = 0$ this gives the harmonic Ritz values. In the symmetric case they provide inner bounds — useful in adaptive methods.

## Expansion and enhancement

- The vector $v$ to be added to $\mathcal{V}$ is generally taken to be a suitably projected solution of a linear system
$$Mv = s.$$
- In Arnoldi $s = v_k$ and $M = A^{-1}$.
- In Jacobi-Davidson $s = Ax - \theta x$.
- Common enhancer is $M = A - \sigma I$. It is related to Newton's method and inverse power method.
- A distinction should be made between the enhancement operator and the preconditioning used to solve the linear system.
- If the system is solved inexactly, the choice $s = r$ has advantages.
- With the generalized eigenvalue problem, some sort of inversion is inevitable.

## Restarting and purging

- Restarting is necessary in large problems owing to storage limitations.
- The purging, like enhancement, steers the algorithm.
- If matrix-products are expensive, this is the place to invest effort.

## Some observations

- The subject needs more theory and fewer algorithms.
- Treasure the people who solve real life problems.
- A common interactive interface would be nice.
- Good progress is being made on generalized eigenvalue problems.
- Unsymmetric problems need more attention.