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THE PROBLEM OF INDUCTION*

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Abstract. This article is the thirtieth of a series of articles discussing various open research problems in automated reasoning. The problem proposed for research asks for criteria for accurately determining when an induction argument is the appropriate form of argument for an automated reasoning program to employ. This research problem also asks for criteria for choosing well the property on which to conduct the induction argument.

Key words. Automated reasoning, induction, unsolved research problem.

Question: *What criteria should be used to determine that an induction argument is appropriate and, if that is the choice, what criteria should be used to select the property on which to base an induction argument?*

(This question is the twenty-ninth of 33 problems proposed for research in [5] and will be referred to as Research Problem 29 throughout this article. All references to sections, chapters, test problems, and such also refer to [5].)

Many proofs from mathematics and from other fields, such as program verification, employ an induction argument. This form of argument is very powerful but is, in the standard interpretation, unfortunately outside the province of first-order predicate calculus. This unfortunate circumstance can be corrected by employment of Gödel's finite axiomatization of set theory (see Section 6.6). This axiomatization gives access to induction within the first-order predicate calculus. Since, for many theorems, the only known proof rests on an induction argument, giving a reasoning program access to induction is clearly a big advantage. The work of Boyer and Moore [1-4] amply supports this observation. Their approach is based not on Gödel's set theory but, instead, on a logic of their own invention.

Research Problem 29 asks for criteria for accurately determining when an induction argument is the appropriate form of argument for an automated reasoning program to employ. This research problem also asks for criteria for choosing well the property on which to conduct the induction argument.

As a research problem not mentioned in [5], we suggest, especially for one interested in implementation, the study of adding induction to McCune's program OTTER [6]. We emphasize OTTER, first because it is widely accepted throughout the world as a powerful, general-purpose reasoning program, and second because this program does not currently offer induction. The addition of induction would make OTTER even more valuable as a research assistant and more useful for various applications such as program verification. The source code (in C) is offered on diskette in [6].

An appropriate test for a proposed solution to Research Problem 29 focuses on problems from program verification and on well-known theorems from mathematics whose proof employs induction. A solution to this problem would be very significant, especially since, for many theorems, an induction argument is the only known type of argument that succeeds.

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References

1. Boyer, R., and Moore, J, *A Computational Logic,* Academic Press, New York (1979).

2. Boyer, R., and Moore, J, 'A mechanical proof of the unsolvability of the halting problem', *Journal of the ACM* **31,** pp. 441-458 (1984).

3. Boyer, R., and Moore, J, 'Proof checking the RSA public key encryption algorithm', *American Mathematical Monthly* **91,** pp. 181-189 (1984).

4. Boyer, R., and Moore, J, 'Proof-checking, theorem-proving, and program verification', pp. 119-132 in *Contemporary Mathematics, Vol. 29, Automated Theorem Proving: After 25 Years,* ed. W. Bledsoe and D. Loveland, American Mathematical Society, Providence, R.I. (1984).

5. Wos, L., *Automated Reasoning: 33 Basic Research Problems,* Prentice-Hall, Englewood Cliffs, N.J. (1988).

6. Wos, L., Overbeek, R., Lusk, E., and Boyle, J., *Automated Reasoning: Introduction and Applications,* 2nd ed., McGraw-Hill, New York (1992).