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DATA STRUCTURE AND PARALLEL DECOMPOSITION CONSIDERATIONS ON A FIBONACCI GRID

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1. INTRODUCTION

The Fibonacci grid, proposed by Swinbank and Purser (1999), provides attractive properties for global numerical atmospheric prediction by offering an optimally homogeneous, geometrically regular, and approximately isotropic discretization, with (as always) only the polar regions requiring special numerical treatment. It is a mathematical idealization, applied to the sphere, of the multi-spiral patterns often found in botanical structures, such as in pine cones and sunflower heads. Computationally, it is natural to organize the domain into zones, in each of which the same pair (or possibly, triplet) of Fibonacci spirals dominate. But the further subdivision of such zones into tiles of a shape and size suitable for distribution to the processors of a massively parallel computer requires careful consideration if the subsequent spatial computations along the respective spirals, especially those computations (such as compact differencing schemes) that involve recursion, can be implemented in an efficient, load-balanced manner without requiring excessive amounts of inter-processor communications. Even when traditional low-order explicit methods are used, care must still be taken.

We show how certain simple properties of the Fibonacci sequence (whose numbers prescribe the multiplicities of the members of the successive families of spirals that make up the grid) may be exploited in the subdivision of grid zones into arrangements of triangular grid tiles, each with two sides coincident with grid lines and a ragged "hypotenuse" approximately aligned east–west.

2. FIBONACCI AND LUCAS NUMBERS

The Fibonacci sequence, F_n, and the related Lucas sequence, L_n, (e.g., Conway and Guy, 1996) for $n \geq 0$, comprise the sets $(0, 1, 1, 2, 3, 5, 8, \ldots)$ and $(2, 1, 3, 4, 7, 11, 18, \ldots)$, respectively. In each sequence, each number is the sum of the two that precede it. A pair of simple formulae, valid also for negative integers, relate F_n and L_n to the golden ratio, $\phi = (\sqrt{5} + 1)/2$:

$$F_n = [\phi^n - (-\phi)^{-n}]/\sqrt{5}, \tag{1}$$

$$L_n = \phi^n + (-\phi)^{-n}. \tag{2}$$

From the generic identity implied by (1) and (2):

$$F_n L_{n+p} = F_{2n+p} - (-)^n F_p, \tag{3}$$

we see that,

$$F_n L_n = F_{2n}, \tag{4}$$

and,

$$F_n L_{n\pm1} = \begin{cases} F_{2n\pm1} - 1, & n \text{ even;} \\ F_{2n\pm1} + 1, & n \text{ odd.} \end{cases} \tag{5}$$

As we show below, these results are particularly useful for tackling the problem of achieving a sytematic domain decomposition of the Fibonacci grid.

3. DOMAIN DECOMPOSITION

As discussed in Swinbank and Purser (1999), the natural quadrilateral computational grid at a given latitude is formed by the mutual intersections of two familes of spirals associated with the two consecutive Fibonacci numbers that quantify the spirals in each family. Since the index of one of these Fibonacci numbers, say F_{2n}, must be even, it is exactly factorable according to (4); thus, this zone of the grid may be evenly subdivided into F_n oblique stripes containing L_n spirals each. The complementary grid family of odd index, $F_{2n\pm1}$, *almost* factorizes into F_n transverse stripes of $L_{n\pm1}$, except for a "dislocation" of one grid unit

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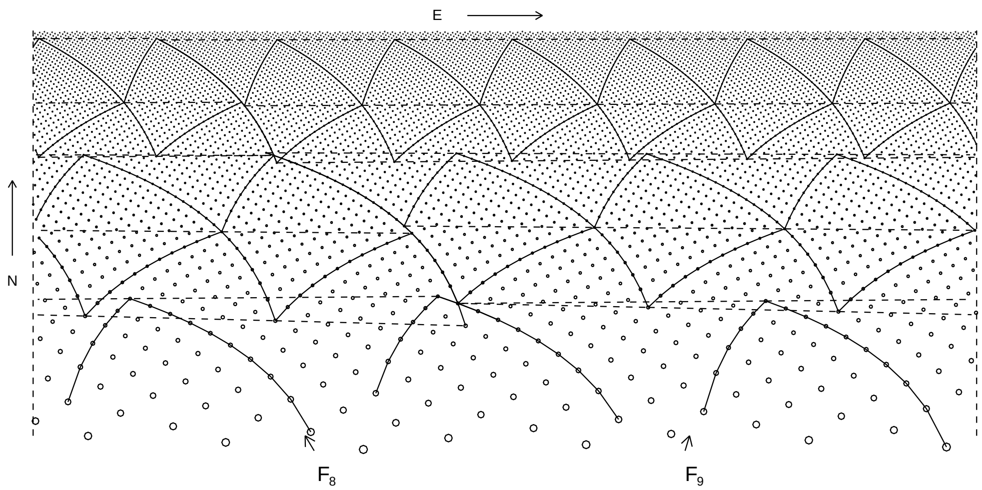

Figure 1. Domain decomposition into triangular tiles of a (southern) circumpolar region of the Fibonacci grid shown in Mercator projection. The representative spirals labeled, "F_8" and "F_9", belong to the families containing $F_8 = 21$ and $F_9 = 34$ members respectively. Note that, for each zone, there is a dual to the decomposition shown, in which the roles of the Fibonacci and Lucas numbers are interchanged.

accumulated in a circuit once around the zone. It is possible to align these dislocations along one common spiral of index F_{2n}. Then, by bisecting the resulting approximately "golden" rectangles by their east-west diagonals, we achieve the desired tiling of this zone by triangles, leaving no gaps or overlaps. From one zone to the next, it is no longer possible to ensure a seemless join but, by arranging a slender overlap in latitude, we ensure that each region of the global grid is covered by our tiling.

To illustrate the kind of pattern obtained, fig. 1 shows the decomposition, as it appears in the conformal Mercator projection, of a portion of the idealized planar version of this grid (the most southerly 30 points are omitted to keep the paper short). In this schematic depiction, each zone contains only a single "course" of triangle-pairs; in practice, the broad equatorial and midlatitude grid zones, accounting for the bulk of the numerical computations, would each contain several courses of identical tile-pairs of the largest dimensions (and therefore the most efficient inter-processor communications). Towards the poles, both the numbers and sizes of the tiles in each zone become progressively smaller, so that efficient load balancing would require, near the poles, a much larger ratio of tiles per processor than near the equator; at sufficiently large latitudes, entire zones would be accommodated in each processor, and the individual tiles could be dispensed with.

4. DISCUSSION AND CONCLUSIONS

We have provided an outline of a generic method of decomposing a Fibonacci grid into smaller units for use in massively parallel computer architectures. The most striking feature of this decomposition is that the elementary tiles are triangular (but such that pairs of them can always be stored in regular rectangular arrays, which simplifies the coding).

The application of general recursive numerical procedures (e.g., compact differencing) always presents a special difficulty in the context of distributed domains, and the apparent extra irregularity incurred by a triangular tiling might suggest that this problem is rendered intractable. However, the recursive operations we encounter on an inherently regular grid can be invariably represented with constant coefficents, and we find that these processes can be accommodated efficiently even when the lengths of segments in each tile are uneven.

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