# A Mathematical Framework for Understanding Continuum Effects of Budget Fluctuations on a University

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As a large part of most state budgets, public universities are tempting targets of budget cuts. Using the theory of diffusion of innovation, we build a mathematical framework that allows us to simulate the continuum effects of such budget cuts. This technique of mathematical modeling can provide insights into the mechanisms that affect the beneficial impact of the university. Such insights are difficult to obtain from standard economic impact studies.

University economic impact studies generally examine the effect of the presence versus the complete absence of an institution. As a result of many of these studies, policymakers tend to agree that the presence of a university is beneficial to the regional economy; see, for example, (Mercer, 1996). Less attention, however, has been paid to the effects of budget cuts or increases on the functioning and economic impact of an existing university.

In this paper, we develop a mathematical framework for studying the continuum effects of persistent budget fluctuations on the university and its region. After briefly discussing the limitations of standard economic impact studies, we present a very condensed review of the literature, which will provide the foundation for the assumptions built into the mathematical framework.

The application of this framework to the university dynamic and the numerical simulation results to which the framework gives rise support the understanding that draining resources from state-funded colleges and universities will eventually diminish the positive regional effects of these institutions. By employing simultaneous differential equations, we demonstrate possible mechanisms by which budget cuts may lead to attenuation of benefit over time.

## UNIVERSITY ECONOMIC IMPACT STUDIES

Today, economic impact studies are commonly used as public relations tools for colleges and universities (Dean, 1991). The usual way to perform an economic impact study is to manufacture a scenario in which an existing university ceases to exist, and then to examine the differences between the university and the no-university scenarios. This approach is illustrated in one definition of economic impact: "We define economic impact as the difference between existing economic activity in a region given the presence of the institution and the level that would have been present if the institution did not exist" (Beck et al., 1995:2).

### *Limitations of Current Economic Impact Studies*

Although the abundance of economic impact studies has made substantial contributions toward understanding the regional benefits of the presence of universities, this form of study does suffer from some limitations.

**Lack of empirical data.** In previous economic impact studies, the contribution of universities to human capital and economic development is acknowledged but not quantified (Beck et al., 1995). Because of the persistent lack of empirical data for higher education, researchers have made contributions to this area by instead appropriately adapting relevant findings from research in other professions, such as medicine. For example, Bess (1998) examined the motivational effects of tenure within the university by adapting findings from the medical profession (Lieberman, 1983; Sitkin & Bies, 1994; Stewart & Cantor, 1982; Van Maanen & Barley, 1984).

**Short time horizon.** The effects of the university on a region are both long-term and short-term (Felsenstein, 1996). Long-term effects, unfortunately, are difficult to measure, since the complete rate of return to education can be assessed only at the end of an individual's lifetime (Quiggen, 1999).

**Inability to model continuous phenomena.** Most important, these economic impact studies tend to be binary in essence, allowing one to compare the impact of an existing institution with the effect if the institution were to be completely removed. As Beck et al. (1995:13) note, "An economic impact study, by its very nature, must always be a comparative analysis." The long-term continuum effects of slowly starving a university of funding and support are not, and cannot be, accounted for in such studies. In the next sections, we discuss what some of those effects might be.

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## BENEFITS OF TERTIARY EDUCATION

Not only is education in general beneficial to regional development (Barro, 1991; Chatterji, 1998; Fedderke & Klitgaard, 1998; Florax, 1992; Lau, Jamison, Liu, & Rivkin, 1993; Lau, Jamison, & Louat, 1990; Lin, 1997; Lipton, 1977; Maddison, 1982; Mankiw, Romer, & Weil, 1992; Marshall, 1890/1930; O'Rourke & Williamson, 1995; Preston, 1997; Psacharopoulos, 1973, 1981; Schultz, 1960, 1961; Smith, 1776/1981), but tertiary education in particular has been found to be an important driver of economic growth. Public investment in university-level education and research has been shown consistently to pay dividends in economic growth and enhanced productivity (Denison, 1968; Feller, 1990; Felsenstein, 1996). University-level education has been found to be more significant in this regard than primary or secondary education (Chatterji, 1998). The exception to this is in underdeveloped countries that lack primary and secondary education and would not benefit from an expansion of the university system until the lower levels of education have been brought up to adequate standards (Kelly, 1997).

### *How a University Fosters Economic Activity*

To date, no comprehensive structural model has linked social institutions and economic growth (Fedderke & Klitgaard, 1998). Individual studies, however, indicate that the presence of an active, effective university benefits the community in several ways.

**The university enlarges the supply of human capital.** Human capital is a major factor that enhances economic growth. Universities not only produce knowledge but also add an "attractiveness value" to the region and confer both short-term and long-term benefits to the region (Bluestone, 1993; Chatterji, 1998; Felsenstein, 1996; Quiggen, 1999; Stern, 1991).

**The university fosters specific skills, technical knowledge, or commercially viable research.** The activity that takes place at a university can encourage investment in a region, which then drives economic growth (Andersson, Anderstig, & Hårsman, 1990; Chatterji, 1998; Florax, 1992; Knack & Keefer, 1997; Lucas, 1988; Mueller, 1989; Romer, 1990; Stenberg, 1990).

**The university brings in economic activity as would any large organization.** Simply because of their size and presence, universities are bound to have some positive effect on economic development (Felsenstein, 1996). The presence of local and out-of-state students further enhances the university's economic impact (Beck et al., 1995).

In the mathematical model we develop, we incorporate the beneficial economic effects of a university by allowing a population of "productive" professors and "successful" students to stimulate regional industrial activity.

The definition of "productivity" or "success" varies with the particular requirements and standards of a given institution. We define productivity and success in relative terms, as actions that fulfill the mission of the institution. For example, productivity in a faculty member could be quantified by accounting for teaching evaluation scores, number of papers published, number of talks given, or number of external grants received in a given year. Productivity, or success, in a student might be quantified by grade point average, standardized test scores, or successful fulfillment of all requirements for graduation. In the mathematical model developed here, we allow a member of a population to be placed in one of two categories: productive (successful) or nonproductive (nonsuccessful). Multiple levels of productivity and success are possible, but since these would significantly increase the complexity of the model, we choose at this point to include only the two categories specified.

We emphasize that merely having a university in an area does not guarantee a fixed amount of benefit to the community. Certain factors affect the magnitude of benefit that the university confers, including an existing local economy in reasonable health and a good reputation outside the region (Felsenstein, 1996).

A healthy regional economy and a successful university are mutually reinforcing. To get this beneficial cycle in motion, the university requires faculty that will enhance the university's reputation through effectively fulfilling the mission of the institution, whether it be research, teaching, or something else. Through faculty activity supported by effective administration, the university's reputation is enhanced. The coordinated, cooperative actions of faculty and of administrative and support staff are crucial. An enhanced reputation built upon effective fulfillment of the institution's mission will attract students from outside the region.

## MOTIVATORS OF UNIVERSITY PRODUCTIVITY

The literature on workplace motivation is vast, so we restrict our discussion here to aspects of motivation that are directly relevant to our model.

### *Adequate Institutional Support*

Herzberg's classic motivation-hygiene theory suggests that employees are most motivated by intangible factors such as achievement, enjoyment of the work itself, recognition, and responsibility (Herzberg, Mausner, & Snyderman, 1959). If these needs are not fulfilled, then motivation will decline, regardless of pay level or tenure (Bess, 1998). Alternatively, employees can be demotivated effectively by perceptions of insufficient pay, inequitable work assignments, inefficient organizational procedures, and inadequate physical facilities (Herzberg et al., 1959; Herzberg, 1987; Robbins, 2000). Herzberg's dissatisfiers, or "hygiene factors," include diminished physical building maintenance, insufficient pay raises, and lack of adequate administrative support. Regular maintenance, pay raises, and administrative support are high on the list of items that are discarded when budgets are cut. As budgets decline, politics loom large as increasingly desperate factions compete for ever-scarcer resources. An increased emphasis on frugality often leads to elaborate tracking and documentation of every penny spent, and consequently to Herzberg's leading workplace dissatisfier: inefficient and frustrating organizational rules (Herzberg, 1987).

If budget cuts are sufficiently large or persistent, it seems reasonable to predict that widespread demotivation, followed by a decrease in effective fulfillment of the institution's mission, will follow. Our mathematical model reflects this by allowing decreases in funding levels to lead to decreased productivity among professors.

### *Availability of Resources*

Apart from the question of demotivation, the lack of necessary resources will negatively affect the ability of university employees to perform their jobs in order to further the university's mission. The lack of adequate resources will constrain performance regardless of motivation or intentions (Blumberg & Pringle, 1982). For example, a professor who no longer has a student helper will need to give up class preparation or research time in order to perform the grading or lab work that the student helper used to cover.

In our mathematical model, we consider a pool of successful students to be a resource for professors, while productive professors are a resource for students. Therefore, we allow for these two populations to simultaneously affect each other, specifying that an increase in these resources will stimulate productivity while a decrease will dampen productive behavior.

### *Actions of Colleagues*

In academia, as in many professions, the opinion of peers and norms of the professional group are more important than formal sanctions and rewards in directing behavior. Peer group standards and the enforcement of those standards by subtle peer pressure constitute the primary means of ensuring compliance to expectations, whether those expectations are for high or low productivity (Bess, 1998). If demotivation leads to changes in effort expended by some individuals, group norms may shift and discourage the output of extra effort by faculty (Comer, 1995).

Our model reflects this peer group effect by allowing productive or successful behavior to stimulate further productive or successful behavior within the respective populations of professors and students.

## PROPOSED MATHEMATICAL MODEL

We propose that the well-known beneficial effects of the university upon the regional economy can be severely compromised by ill-thought-out budget cuts. Continually diminishing financial resources to the university will, we believe, result in reduced benefit to the community.

Through the use of simultaneous continuous differential equations, we seek to address some of the limitations of existing research. In particular, the use of a mathematical model of this nature will allow us to simulate continuous, as opposed to binary, phenomena. Additionally, simulations can be carried out over long time horizons.

Unfortunately, as with previous studies, the problem of lack of empirical data persists. However, when possible, relevant observations in the literature have been translated into core model elements. The model should be viewed as a framework for analysis. Into this framework can be placed coefficients and parameter values tailored to reflect the specific situation under study.

### *The University Model - Overview*

In the model we develop, we simulate the peer group effect of success breeding success within a population by employing mechanisms that are similar to those used to describe the diffusion of technology and innovation. Mansfield (1961) introduced a mathematical model of diffusion of innovation in the context of studying how rapidly the use of a number of innovations spread from enterprise to enterprise in several separate industries. For a simple description of the mathematical model of diffusion of innovation, see, for example, (Braun, 1978: 37-43) .

Diffusion of innovation models can be appropriately used to describe any situation in which the development, implementation, and dissemination of new ideas, behaviors, methods, or products in a business, an organization, or society as a whole are of interest. For example, Strang and Soule (1998) discuss the application of diffusion of innovation to individuals and the factors motivate individuals to adopt certain behaviors.

In our mathematical model, "productivity" and "success" are behaviors that can be diffused by individual adoption throughout the university organization. The model we create is then built on a mathematical description of the diffusion of productivity or success and on factors that can either accelerate or dampen the rates at which such diffusion takes place.

The simultaneous dependencies in the model we propose are represented graphically in Figure 1. Note, in particular, that we incorporate the influence of funding levels on the population of professors, with a feedback effect of the populations onto themselves as well as onto levels of external grant funding. We also incorporate the assumption that the student and professor populations will mutually affect each other and, in turn, affect regional industry.

### *The University Model - Equations*

When building any mathematical model, we believe that it best to start simply and to add complexity only when called for. The model we present can be considered relatively simple, tracking only three interacting populations: professors, students, and the regional industrial jobs. We consider how funding levels affect the productivity of the professor population, and we make fairly straightforward assumptions about causalities.

We first introduce the variable terms that will be needed for the model.

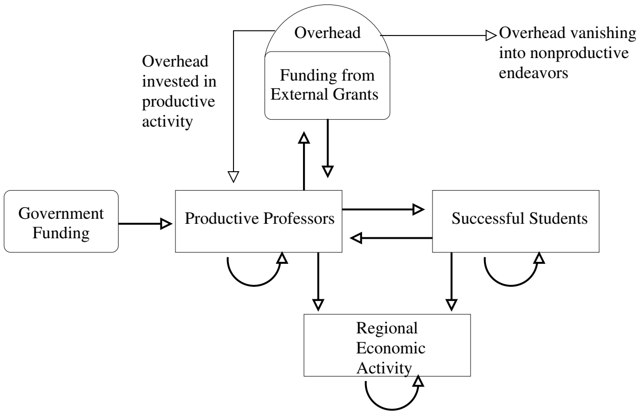


*Figure 1.* Simultaneous dependency flow chart for mathematical model of a university dynamic.

• $P(t)$ = total number of "productive" professors at time $t$. At this stage we assume that a professor is either productive or not. We have not incorporated degrees of productivity, and we have not specified the precise measure of productivity. As discussed above, any quantifiable measure of productivity consistent with the mission of the institution can be used.

• $P_T$ = total number of professors at the university. We are assuming that we are not trying to increase the total number of professors at this point.

• $S(t)$ = number of successful students enrolled at the university at time $t$.

• $S_T$ = fixed total student population.

• $H(t)$ = number of industrial positions in the region at time $t$.

• $H_T$ = saturation point, beyond which the total number of industry jobs can no longer increase. We link this saturation point $H_T$ directly to the population in the region. In this case, we choose it to be a multiple of the total of professors and students together, so that $H_T = m(P_T + S_T)$, where $m$ is a positive factor.

• $I_P$ = average number of grant dollars that a productive professor is able to procure in a year. One outgrowth of the kinds of endeavors that are commonly considered productive is the procurement of external grant funding. Since this is a readily quantifiable measure of productivity, we choose this to be a feature associated with productive behavior in this model. We are not specifying the ability to bring in grant dollars as a cut-off measure of productivity. We are simply making the assumption, for this model, that on average this class of professors will bring in $I_P$ external grant dollars per professor per year.

• $E_P$ = average amount of money it takes to provide basic support one full-time professor for one year.

• $E_T$ = total amount of funding the university needs to maintain the current faculty population for one year. In this case, $E_T = P_T E_P$.

• $I_E(t)$ = number of external grant dollars available at time $t$. Since we assume that each productive faculty member is able to produce, on average, $I_P$ external grant dollars, and we have $I_E(t) = I_P P(t)$.

• $O_T$ = fraction of overhead taken out of faculty grants and claimed by the university administration. We require that $0 \leq O_T \leq 1$.

• $\alpha$ = fraction of overhead, $O_T$, reinvested by the administration to support productive activities in the faculty population. We require that $0 \leq \alpha \leq 1$.

• $F(t)$ = amount of external grant funding used to support productive activities in the professor population. Here $F(t) = (1 - O_T + \alpha O_T) I_E(t)$. The term $I_E(t) = I_P P(t)$ represents the total number of external grant dollars available at time $t$, and the coefficient $(1 - O_T + \alpha O_T)$ represents the fraction of those grant dollars that are used directly to support productive endeavors.

• $I_G$ = total income to the university from government funding per year.

The change over time in the size of the population of either productive professors or successful students, can be thought of as happening in one of two ways. Either a single individual can leave the nonproductive (nonsuccessful) population and become part of the productive (successful) population, or a nonproductive (nonsuccessful) individual may leave the system altogether and be replaced by a new productive (successful) individual. Mathematically, both occurrences can be described in the same way: when one population loses an individual, the complementary population gains an individual. Of course, this is allowed because of the assumption of constant total population size. Future model refinements will allow for fluctuations in total population size, and the mathematical description will become more complicated.

The change over time in the interacting populations of productive professors, successful students, and regional industry positions is described by the following system of differential equations:

$$\frac{dP}{dt} = c_0 \left( \frac{(c_1(I_G - E_T) + c_2 F)}{E_P} + c_3 S - c_4 (S_T - S) \right) R \quad (1)$$

$$\text{with } R = \left( \frac{P}{P_T} \right) (P_T - P)$$

$$\frac{dS}{dt} = d_0 \left( d_1 P - d_2 (P_T - P) \right) \left( \frac{S}{S_T} \right) (S_T - S) \quad (2)$$

$$\frac{dH}{dt} = e_0 H \left( 1 - \frac{H}{H_T} \right) \left( e_1 \frac{\beta P - P_T}{P_T} + e_2 \frac{\gamma S - S_T}{S_T} \right) \quad (3)$$

where coefficients $c_0, c_1, c_2, c_3, c_4, d_0, d_1, d_2, e_0, e_1$, and $e_2$ are all positive scaling parameters.

The fundamental form of Equations (1) and (2) is that of a basic mathematical diffusion of innovation model. In general, diffusion of innovation models have logistic solutions. This means that the process of innovation adoption accelerates to a point and then decelerates as the innovation begins to saturate the community. Equations (1) and (2) therefore reflect the dynamic that productivity will diffuse throughout the population in a logistic manner.

In Equation (1), the term $\left( \frac{P}{P_T} \right) (P_T - P)$ represents the assumption that the number of professors who convert from being nonproductive to being productive is directly proportional to the respective populations of productive and nonproductive professors. Similarly for Equation (2), the term $\left(\frac{S}{S_T}\right)(S_T - S)$ represents the assumption that the successful student population grows proportionally to the populations of successful and nonsuccessful students.

Through term $\left(\frac{(c_1(I_G - E_T) + c_2 F)}{E_P} + c_3 S - c_4(S_T - S)\right)$ of Equation (1), we incorporate the effect of funds that are made available to encourage productive activity, as well as the effect of the student population on the professors. This allows for $P$ to be positively affected by any amount of government funding, $I_G$, that exceeds the minimum necessary expenditures, $E_T$, as well as by the availability of external grant funds that are being put toward the development and maintenance of productive professors. On the other hand, if $I_G$ drops below $E_T$, this will negatively impact the growth of $P$. The scaling by $1/E_P$ converts the units from dollar amounts to units of full-time professors. Additionally, $P$ is positively affected by the presence of successful students and negatively impacted by the presence of nonsuccessful students. An example of a way in which positive student effect on the professor population might evidence itself could be in the form of the availability of a qualified pool of student research and teaching assistants. Poor students could negatively affect professor productivity in that poor students may slow the progress of a course, consume institutional resources by filing grievances over poor grades, and in sufficient numbers require the addition of remedial courses to the university curriculum.

In Equation (2), we allow for a mechanism by which the presence of productive faculty will positively influence the population of successful students and nonproductive faculty will negatively influence the population. For example, we might assume that a successful student is attracted to the university by its reputation and that the university's reputation is directly linked to the productivity of its faculty. We might also assume that a student who is already enrolled and who has the potential to become successful can be influenced to success by active faculty, whereas disengaged faculty can even drive a successful student toward becoming unsuccessful. The influence of faculty on the student population is represented by the term $(d_1 P - d_2(P_T - P))$.

Equation (3) describes the change over time in the number of industrial positions ($H$) available in the university region. As indicated by the dependencies graph in Figure 1, the equations for $P$ and $S$ will not be directly affected by $H$. This reflects the assumption that the presence of industry in a university town does not significantly affect whether a professor is productive or whether a successful student will choose to attend that university. On the other hand, we assume that the presence of successful students in the area, as well as the availability of productive professors, will positively influence the growth of $H$. The implicit assumption here is that industry jobs are those requiring college degrees, directly fillable by university graduates. Additionally, we assume that $H$ is positively affected by the number of other industrial positions currently in the region and that there is saturation point, $H_T$, beyond which the market can no longer grow.

The term $e_0 H$ says that $H$ grows proportionally to itself (i.e., if there are already industrial jobs in the area, it will attract more industrial jobs). This term is multiplied by $(1 - H/H_T)$, which says that there can be saturation in the market. That is, once we start getting near to having $H_T$ jobs in the region, the growth in the number of industrial positions will slow and will level off at $H_T$ jobs. The last two terms, $(\beta P - P_T)/P_T$ and $(\gamma S - S_T)/S_T$, say that if the *productive* professor population drops below the fraction $1/\beta$ of the *total* professor population, and if the *successful* student population drops below the fraction $1/\gamma$ of the *total* student population, there will be a negative impact on the growth in the number of industrial positions in the region.

The system of Equations (1), (2), and 3 is a mathematical framework that now can be employed to simulate the interdependent continuum effects of modifying budget levels on a university. Particular scenarios will differ depending on parameter sets chosen and on initial sizes of the three populations being tracked.

## *The University Model - Numerical Simulations*

In this section we present an example of numerical solutions of the system (1), (2), and (3). We first discuss our choice of parameter values. We remind the reader that in the literature, there is a lack of relevant empirical data from which we can derive precise model parameters.

In lieu of precise measurements, therefore, we model a hypothetical university using parameters that make intuitive sense and that give rise to natural results. Experimentation indicates that incorporation of different parameter sets allows the fundamental qualitative behavior of the model to remain intact, while quantitative outcomes will vary. That is, the trends implied by the computational results will continue to follow logistic paths in all simulations, but exact numerical quantities and how rapidly those quantities change over time will differ.

As indicated earlier, some behaviors and outcomes are best observed with longer time scales. Hence, we run our simulation over a fifty-year time interval, which should enable us to observe certain long-term trends.

We choose our hypothetical university to be of moderate size. We simulate the evolution of the system in time given two different sets of initial conditions. In the first case, we assume that our university starts out with a somewhat weak profile: one-quarter of the professors will be categorized as productive and one quarter of the students as successful. In the second case, we strengthen our university's initial profile and allow half of the professors to be productive and half of all students to be successful.

We track the progression of the three interdependent populations, $P(t)$, $S(t)$, and $H(t)$, in our model using the following parameter values: $P_T = 1000$, $S_T = 15000$, $H_T = 3(P_T + S_T)$, $I_P = \$150,000$, $E_P = \$100,100$, $O_T = 0.25$, and $\alpha = 0.1$.

In the case that the university begins with a relatively weak profile, we have $P(0) = 250$ and $S(0) = 3750$. We also choose the initial number of industry jobs to be proportional to the initial university population, so that $H(0) = 3(P(0)+S(0))$. In the case that the university begins with a somewhat strengthened profile, we double our initial productive population numbers, which then become $P(0) = 500$ and $S(0) = 7500$.

Additionally, we let $1/\beta = 1/2$ and $1/\gamma = 2/3$. These parameters imply that if the productive professor population drops below 50% of the total professor population and if the successful student population drops below 66% of the total student population, the number of industrial jobs in the region will be negatively affected. We note that our initial productive professor populations and successful student populations are each at 25% (weak case) or at 50% (strong case) of the total. Hence, in the weak university case, we would expect to see an initial decline $H$, at least until the successful student and productive professor populations can achieve their critical thresholds. In the strong university case, the threshold is already met for professors, but the student population falls just short. In our model parameters, we have chosen the influence of good students on industry to be somewhat stronger than that of productive professors, and we will see this reflected in the simulations.

We assume that dollar amounts are implicitly adjusted for inflation and therefore do not include explicit terms to account for possible changes in the value of the dollar. We examine three cases:

1. Maintaining the same levels (relative to inflation) of government funding over the years.
2. Decreasing government funding by 5% per year (relative to inflation).
3. Increasing government funding by 5% per year (relative to inflation).

In each case we modify only the parameter that affects the level of government funding. All other parameters remain the same.

## RESULTS

In each simulation, the effects of budget cuts and increases are not immediately observable, but can be clearly seen over longer time frames.

**Maintaining government funding.** In Figures 2 and 3 we see the evolution over time in the three populations when government funding to the university is maintained. The simulations indicate that with an initially weak profile, steady government funding will allow the population of productive professors to grow, albeit quite slowly. Unfortunately, the growth in the professor population is not sufficiently rapid to stem the decline both in the student population and in industry. On the other hand, with an initially strong profile and unwavering levels of government support, both the productive professor and student populations will increase over time. After suffering an initial decline, industry will eventually increase. Note, however, that industry will not begin to increase significantly until the necessary successful student population threshold is crossed.

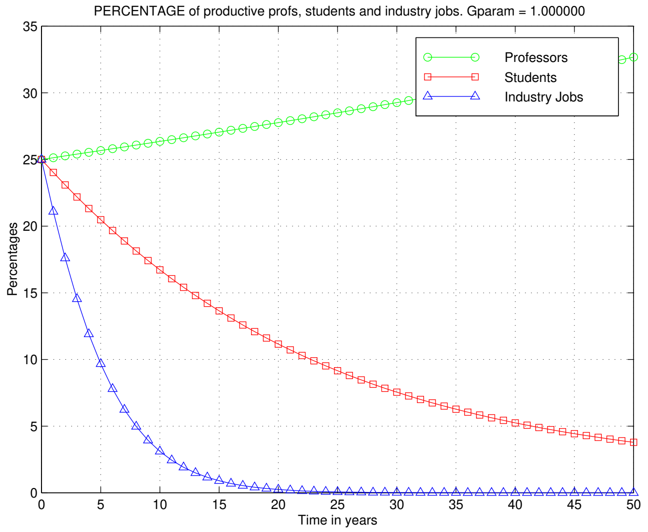


*Figure 2.* Initially weak university profile: Effect of maintaining levels of government funding from year to year.

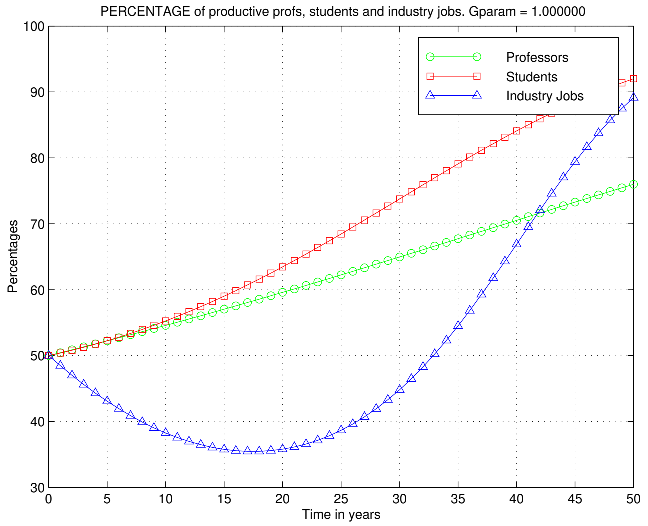


*Figure 3.* Initially strong university profile: Effect of maintaining levels of government funding from year to year.

**Decreasing government funding.** The effects of decreasing government funding can be seen in Figures 4 and 5. Unsurprisingly, all three populations suffer decline over the years. It is interesting to note, however, that with a stronger initial profile, the productive professor population stays level for about five years before turning downward. The successful student population also manages to maintain positive momentum, increasing for a number of years, even in the face of budget cuts. Nonetheless, the budget decreases eventually catch up with the students as well, and all populations eventually diminish.

**Increasing government funding.** We see a more optimistic picture in Figures 6 and 7. These reflect the outcome

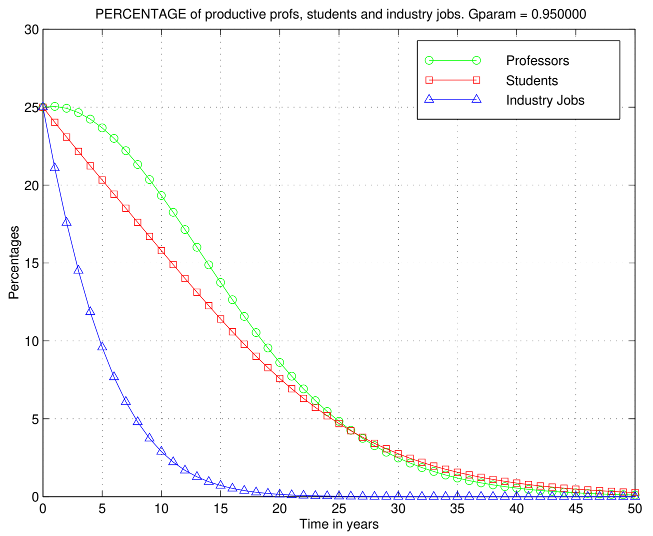


*Figure 4.* Initially weak university profile: Effect of decreasing government funding by 5% each year.

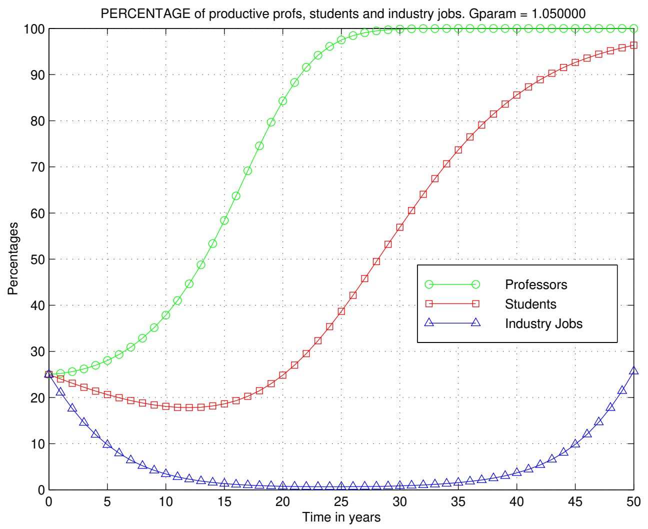


*Figure 6.* Initially weak university profile: Effect of increasing government funding by 5% each year.

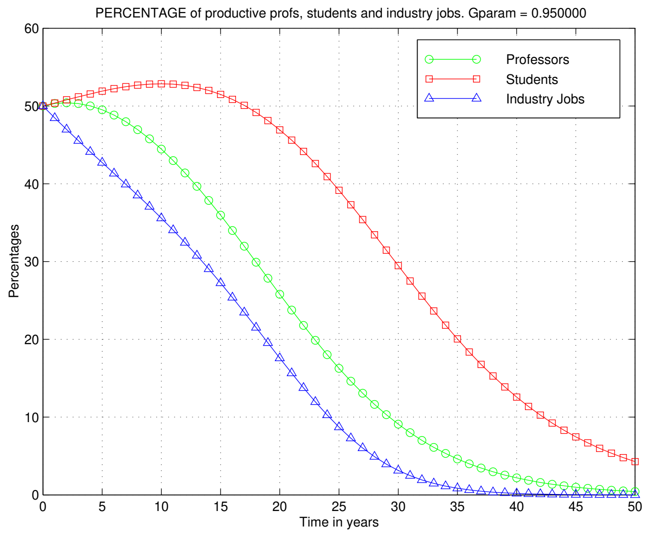


*Figure 5.* Initially strong university profile: Effect of decreasing government funding by 5% each year.

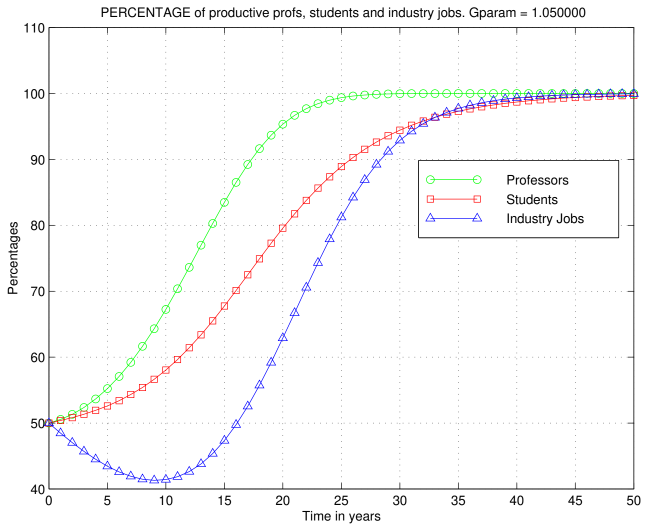


*Figure 7.* Initially strong university profile: Effect of increasing government funding by 5% each year.

when government funding is steadily increased by 5% each year. In both profiles, the professor population benefits significantly from an increasingly strong budget. Additionally, although we see an initial decline in industry, an upswing occurs eventually in both cases. The main difference can be seen in the time lag between the increase in the professor population and that of the student population and industry. With an initially weak university profile, industry does not begin to turn upwards until thirty-five years into the simulation, and it takes fifty years to get back to its initial levels. On the other hand, with an initially strong university profile and steadily increasing government support, industry begins to grow after only ten years, surpasses its starting levels after a little over fifteen years, and hits 80% saturation in about twenty-five years.

## DISCUSSION

We have presented a mathematical framework through which the continuous temporal dynamics of a university organization can be simulated. We adhere to the philosophy that it is best to build a mathematical model by starting simply, adding only those components needed to allow for situation specific fine-tuning and to create outcomes that, if they cannot be compared with collected data, at least make intuitive sense.

The parameter sets chosen for use in this model have largely been determined from general assumptions, which in turn arose from evidence in the literature. Exact data and measurements for use in this model are unfortunately lacking. What the simulation results tell us is that the trends observed are possible but may vary depending on the parameter set reflective of a particular institution. To be able to apply this analysis framework to an actual university organization, we would need to use organization-specific population levels and to conduct investigations to determine precise model parameters.

The results that have come out of this investigation can be useful in directing future research endeavors. We note that any additions to the fundamental structure of the model are likely to make the model far more complex. Nonetheless, we present some possible model extensions.

It was implicitly assumed in the mathematical model that money budgeted toward the encouragement of productive activity was well placed and effectively used by some measure. In future refinements of this model, we plan to examine this assumption more closely, focusing on how funding is apportioned within a university system and what the known outcomes of that apportionment are.

In the current model, the respective populations of professors and students are categorized as either productive or not, successful or not. We plan to explore the possibility of allowing for multiple levels of productivity and success, or even a continuum of such. Such a variation in levels of productivity and success is more reflective of a true university population.

The model presented here incorporated the assumption that total populations of students and professors were unchanging. However, significant growth in the number of students enrolled and the number of professors employed at a university could, in and of itself, be interpreted as a measure of the success of that university. A future extension of this model will allow for fluctuations in total population sizes.

In this model we did not consider the financial impact of the presence of students. The degree of financial impact of the student population varies widely from university to university. In another model refinement, it would be possible to include the effects of tuition income, of raising and lowering tuition levels, and of providing student financial aid as well as merit scholarships, and how these decisions in turn impact the student population and diversity.

It would also be of interest to examine the effects of activities not directly tied to the teaching and research missions of the university, such as athletics, community outreach projects, and continuing education.

The fundamental mathematical framework developed in this paper, while instructive in its own right, can serve well as a foundation from which to build models that are even richer in detail and that allow for an even greater degree of fine-tuning. We believe that the use of this deterministic mathematical approach, in combination with collected data, will allow policymakers to develop well-informed budgeting level and apportionment plans tailored for long-term societal benefit.

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